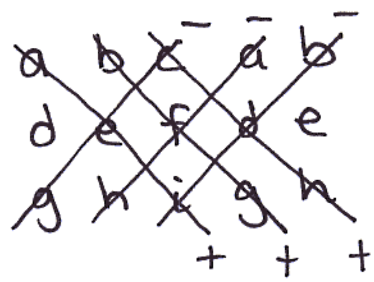
Formula for 3x3 inverses

Dave Bayer

Like the hand method for 3x3 determinants, there is a hand method for 3x3 inverses, which is often quicker and more reliable than either row reduction or the general formula for inverses.

Recall the hand method for 3x3 determinants:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

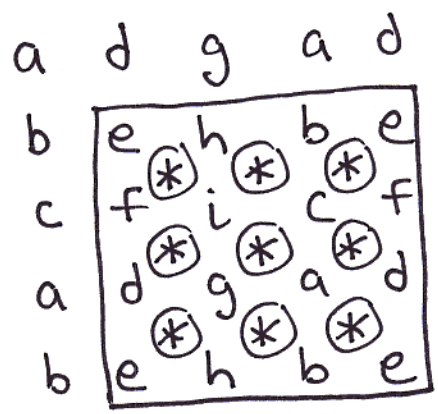


$aei + bfg + cdh$
$- ceg - afh - bdi$

1. Copy original matrix
2. Copy over the first two columns, on the right
3. Multiply along each \ line of 3 terms, with a +
4. multiply along each / line of 3 terms, with a −

The inverse hand method is similar:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$



$$\begin{bmatrix} \circledast & \circledast & \circledast \\ \circledast & \circledast & \circledast \\ \circledast & \circledast & \circledast \end{bmatrix} \Big/ \det$$

Pattern for 3x3 hand inverses, each $\circledast$ is a 2x2 det,

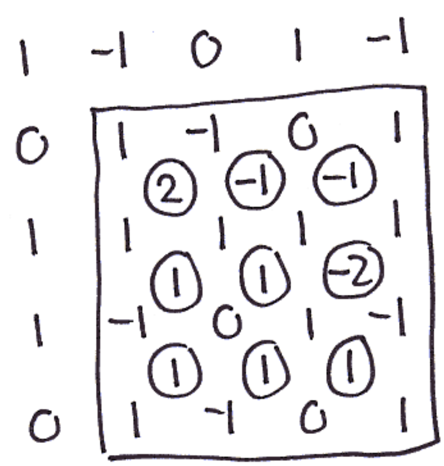
e.g.

$\circledast = ei - fh$

① *Transpose* original matrix, leaving plenty of space.
② Copy over the first two columns, on the right
③ Copy over the first two rows, below
④ Draw a box around all but first row and column.
⑤ In the 9 spaces between 2×2 blocks of entries, write the 2×2 determinant, and circle it.
⑥ Copy out these numbers as the inverse matrix. Divide by determinant of original matrix, which you figure out while checking answer.

Example:

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}}_{\text{matrix}} \Rightarrow$$



$$\Rightarrow \underbrace{\begin{bmatrix} 2 & -1 & -1 \\ 1 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix} \Big/ 3}_{\text{inverse}}$$

(when we begin to check answer, we get $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ as product, so we know det = 3, and dividing by 3 will give the inverse.)

I made this up, but it undoubtedly has been discovered many times. If anyone finds a reference for this method, please tell me.