Exam 2

Linear Algebra, Dave Bayer, March 6, 2014

[1] Find the row space and the column space of the matrix

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 3 & 6 & 9 & 2 \\ 0 & 4 & 8 & 2 & 6 \end{bmatrix}$$

[2] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

[3] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + 3y - 2z \;=\; 0$$

[4] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1,1,0,0) \qquad (0,1,1,0) \qquad (0,0,1,1) \qquad (1,2,1,0) \qquad (0,1,2,1)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[5] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of polynomials of degree $\leqslant 1$. Find the orthogonal projection of the polynomial x^2 onto the subspace W, with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)g(x)\,dx$$