Exam 2
Linear Algebra, Dave Bayer, March 6, 2014

Name: ___________________________________________  Uni: ____________


If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] Find the row space and the column space of the matrix

\[
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 \\
0 & 2 & 4 & 6 & 8 \\
0 & 3 & 6 & 9 & 2 \\
0 & 4 & 8 & 2 & 6
\end{bmatrix}
\]
[2] By least squares, find the equation of the form \( y = ax + b \) that best fits the data

\[
\begin{bmatrix}
  x_1 & y_1 \\
  x_2 & y_2 \\
  x_3 & y_3 \\
  x_4 & y_4 \\
\end{bmatrix} = \begin{bmatrix}
  0 & 1 \\
  1 & 2 \\
  2 & 1 \\
  3 & 1 \\
\end{bmatrix}
\]
[3] Find the $3 \times 3$ matrix that projects orthogonally onto the plane

$$x + 3y - 2z = 0$$
[4] Find an orthogonal basis for the subspace $V$ of $\mathbb{R}^4$ spanned by the vectors

$$\begin{align*}
(1, 1, 0, 0) & \quad (0, 1, 1, 0) & \quad (0, 0, 1, 1) & \quad (1, 2, 1, 0) & \quad (0, 1, 2, 1)
\end{align*}$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$. 
[5] Let $V$ be the vector space of all polynomials of degree $\leq 2$ in the variable $x$ with coefficients in $\mathbb{R}$. Let $W$ be the subspace of polynomials of degree $\leq 1$. Find the orthogonal projection of the polynomial $x^2$ onto the subspace $W$, with respect to the inner product

$$\langle f, g \rangle = \int_{0}^{1} f(x)g(x) \, dx$$