Practive Exam 2

Linear Algebra, Dave Bayer, October 28, 1999

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

To be graded, this practice exam must be turned in at the end of class on Thursday, November 4. Such exams will be returned in class on the following Tuesday, November 9. Participation is optional; scores will not be used to determine course grades. If you do participate, you may use your judgement in seeking any assistance of your choosing, or you may take this test under simulated exam conditions.

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let P be the set of all polynomials $f(x)$, and let Q be the subset of P consisting of all polynomials $f(x)$ so $f(0) = f(1) = 0$. Show that Q is a subspace of P.

Continued on page: ______

Exam 2

Linear Algebra, Dave Bayer, November 11, 1999

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let P be the set of all degree ≤ 4 polynomials in one variable x with real coefficients. Let Q be the subset of P consisting of all odd polynomials, i.e. all polynomials $f(x)$ so $f(-x) = -f(x)$. Show that Q is a subspace of P. Choose a basis for Q. Extend this basis for Q to a basis for P.

 Continued on page: ______

[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Let $W \subset V$ be the set of all polynomials f in V which satisfy $f'(1) = 1$. Is W is a subspace of V? Why or why not?

Continued on (back of) page: ______

Final Exam

Linear Algebra, Dave Bayer, May 8, 2001

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Let $W \subset V$ be the subpace of all *odd* polynomials f in V, i.e. those polynomials $f(x)$ in V which satisfy $f(-x) = -f(x)$. Find a basis for W. Extend this basis to a basis for V.

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answer:

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work:

Continued on back of page ______

[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Find a basis for the subspace W defined by $f(0) = f(1) = f(2)$. Extend this basis to a basis for V.

Continued on (back of) page: ______

[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Find a basis for the subspace W defined by

$$f(-1) = f(0), \quad f(-1) = f(1), \quad f(0) = f(1).$$

Extend this basis to a basis for V.

Continued on (back of) page: ______

[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Find a basis for the subspace W defined by

$$f(1) = f'(1) = 0.$$

Extend this basis to a basis for V.

Continued on (back of) page: ______

[2] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 3$. Find a basis for the subspace W defined by

$$f(x) = f(-x)$$

Extend this basis to a basis for V.

[**4**] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 3$. Find a basis for the subspace W defined by

$$f(0) = f(1) = f(2)$$

Extend this basis to a basis for V.

Exam 2, 11:00-12:15

Linear Algebra, Dave Bayer, March 27, 2012

[1]	[2]	[3]	[4]	[5]	**Total**

Please draw a box around your final answer. Please use each printed sheet (front and back) *only* for that problem, not for any other problem. There are blank sheets at the end of the exam, to give you more room to work. However, your final answer will not be graded unless it appears on the same sheet (front or back) as the printed problem.

[1] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 2$. Find a basis for the subspace W defined by

$$f'(1) = 0.$$

Extend this to a basis for V.

Exam 2, 9:10-10:25

Linear Algebra, Dave Bayer, March 27, 2012

Name: ______________________________ **Uni:** ______________

[1]	[2]	[3]	[4]	[5]	Total

Please draw a box around your final answer. Please use each printed sheet (front and back) *only* for that problem, not for any other problem. There are blank sheets at the end of the exam, to give you more room to work. However, your final answer will not be graded unless it appears on the same sheet (front or back) as the printed problem.

[1] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 2$. Find a basis for the subspace W defined by

$$f(1) = 0.$$

Extend this to a basis for V.

[2] Let A be the matrix

$$A \;=\; \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 1 & -1 & 0 \end{bmatrix}.$$

Compute the row space and column space of A.

Continued on page: ______

[2] Let A be the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}.$$

Compute the row space and column space of A.

Continued on page: ______

Exam 2

Linear Algebra, Dave Bayer, March 29, 2001

Name: ____________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}.$$

Compute the row space and column space of A.

 Continued on (back of) page: ______

Exam 2

Linear Algebra, Dave Bayer, April 3, 2003

Name: ______________________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & -3 & 5 & -7 \\ -1 & 2 & -3 & 5 & -7 & 12 \\ 2 & -3 & 5 & -7 & 12 & -19 \end{bmatrix}.$$

Compute the row space and column space of A.

 Continued on (back of) page: ______

Second Midterm

Linear Algebra, Dave Bayer, March 30, 2004

Name: ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ -1 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 1 & -2 & 1 \end{bmatrix}.$$

Compute the row space and column space of A.

Continued on (back of) page: ______

Second Midterm

Linear Algebra, Dave Bayer, March 29, 2005

Name: ______________________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 1 & 2 & 3 & 5 \\ 1 & 1 & 2 & 3 & 5 & 8 \\ 1 & 2 & 3 & 5 & 8 & 13 \\ 1 & 3 & 5 & 8 & 13 & 21 \end{bmatrix}.$$

Compute the row space and column space of A.

 Continued on (back of) page: ______

[3] Find a basis for the rowspace, and find a basis for the column space, of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 & 2 & 0 \\ 2 & 0 & 1 & 2 & 0 & 1 \end{bmatrix}$$

Continued on (back of) page: ______

[3] The four vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

span a subspace V of $\mathbb{R}^3$, but are not a basis for V. Choose a subset of $\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3,\ \mathbf{v}_4\}$ which forms a basis for V. Extend this basis for V to a basis for $\mathbb{R}^3$.

Continued on page: ______

[**2**] Let

$$\mathbf{v}_1 = (1, 1, 0, -1), \quad \mathbf{v}_2 = (1, 0, 1, -1), \quad \mathbf{v}_3 = (0, 1, 1, -1), \quad \mathbf{v}_4 = (1, -1, 0, 0).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

Continued on (back of) page: ______

[2] Let

$$\mathbf{v}_1 = (1, 2, -3, -4), \quad \mathbf{v}_2 = (1, -2, 3, -4), \quad \mathbf{v}_3 = (0, 2, -3, 0), \quad \mathbf{v}_4 = (1, -2, -3, 4).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$. Extend this basis to a basis for $\mathbb{R}^4$.

 Continued on (back of) page: ______

[**2**] Let

$$\mathbf{v}_1 = (1,1,0,0),\ \mathbf{v}_2 = (1,0,1,0),\ \mathbf{v}_3 = (1,0,0,-1),\ \mathbf{v}_4 = (0,1,-1,0),\ \mathbf{v}_5 = (0,1,0,1).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, and $\mathbf{v}_5$. Extend this basis to a basis for $\mathbb{R}^4$.

 Continued on (back of) page: ______

[4] Find a basis for the nullspace (homogeneous solutions) of the matrix shown. Extend this basis to a basis for all of $\mathbb{R}^5$.

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

 Continued on (back of) page: ______

[2] Find a basis for the subspace V of $\mathbb{R}^4$ defined by the following system of equations. Extend this basis to a basis for all of $\mathbb{R}^4$.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[2] Find a basis for the subspace V of $\mathbb{R}^5$ defined by the following system of equations. Extend this basis to a basis for all of $\mathbb{R}^5$.

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Exam 2

Linear Algebra, Dave Bayer, March 29, 2011

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] Find a basis for the rowspace of the following matrix. Extend this basis to a basis for all of $\mathbb{R}^4$.

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

[5] Find a basis for the subspace V of $\mathbb{R}^4$ spanned by the rows of the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 6 \end{bmatrix}.$$

Extend this basis to a basis for all of $\mathbb{R}^4$.

[5] Find a basis for the subspace V of $\mathbb{R}^4$ given by the equation $w + x + y + 2z = 0$. Extend this basis to a basis for all of $\mathbb{R}^4$.

[4] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which rotates one half turn around the axis given by the vector $(1, 1, 1)$. Find a matrix A representing L with respect to the standard basis

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Choose a new basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$ which makes L easier to describe, and find a matrix B representing L with respect to this new basis.

Continued on page: ______

[5] Let $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ and $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ be ordered bases for $\mathbb{R}^2$, and let L be the linear transformation represented by the matrix A with respect to $\{\mathbf{e}_1,\ \mathbf{e}_2\}$, where

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad A = \begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}.$$

Find the transition matrix S corresponding to the change of basis from $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$. Find a matrix B representing L with respect to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$.

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[**3**] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which reflects through the plane P defined by $x + y + z = 0$. In other words, if $\mathbf{u}$ is a vector lying in the plane P, and $\mathbf{v}$ is a vector perpendicular to the plane P, then $L(\mathbf{u} + \mathbf{v}) = \mathbf{u} - \mathbf{v}$. Choose a basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$, and find a matrix A representing L with respect to this basis.

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[4] Let $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ and $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ be ordered bases for $\mathbb{R}^2$, and let L be the linear transformation represented by the matrix A with respect to $\{\mathbf{e}_1,\ \mathbf{e}_2\}$, where

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 2 \\ -4 & 5 \end{bmatrix}.$$

Find the transition matrix S corresponding to the change of basis from $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$. Find a matrix B representing L with respect to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$.

Continued on page: ______

[5] Let $\{\mathbf{u}_1,\ \mathbf{u}_2\}$, $\{\mathbf{v}_1,\ \mathbf{v}_2\}$, and $\{\mathbf{w}_1,\ \mathbf{w}_2\}$ be ordered bases for $\mathbb{R}^2$. If

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

is the transition matrix corresponding to the change of basis from $\{\mathbf{u}_1,\ \mathbf{u}_2\}$ to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$, and

$$B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

is the transition matrix corresponding to the change of basis from $\{\mathbf{u}_1,\ \mathbf{u}_2\}$ to $\{\mathbf{w}_1,\ \mathbf{w}_2\}$, express $\mathbf{v}_1$ and $\mathbf{v}_2$ in terms of $\mathbf{w}_1$ and $\mathbf{w}_2$.

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Final Exam

Linear Algebra, Dave Bayer, December 16, 1999

Name: ____________________

ID: ____________________ **School:** ____________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	
[**5**] (5 pts)	[**6**] (5 pts)	[**7**] (5 pts)	[**8**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which projects onto the line $(1, 1, 1)$. In other words, if $\mathbf{u}$ is a vector in $\mathbb{R}^3$, then $L(\mathbf{u})$ is the projection of $\mathbf{u}$ onto the vector $(1, 1, 1)$. Choose a basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$, and find a matrix A representing L with respect to this basis.

 Continued on page: ______

[4] Let $\mathbf{v}_1 = (1, 2)$ and $\mathbf{v}_2 = (1, 3)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = \mathbf{v}_1, \quad L(\mathbf{v}_2) = \mathbf{v}_1 + \mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$.

 Continued on (back of) page: ______

[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y = z$, and $L(\mathbf{v}) = \mathbf{0}$ for all $\mathbf{v}$ belonging to the subspace $W \subset \mathbb{R}^3$ defined by $x = y = z$. Find a matrix that represents L with respect to the usual basis

$$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1).$$

Continued on (back of) page: ______

[4] Let $\mathbf{v}_1 = (1,1)$ and $\mathbf{v}_2 = (1,2)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = \mathbf{v}_1 + \mathbf{v}_2, \quad L(\mathbf{v}_2) = \mathbf{v}_1 - \mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1,0)$, $\mathbf{e}_2 = (0,1)$.

 Continued on (back of) page: ______

[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y = 2z$, and $L(\mathbf{v}) = 2\mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $W \subset \mathbb{R}^3$ defined by $x = y = 2z$. Find a matrix that represents L with respect to the usual basis

$$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1).$$

 Continued on (back of) page: ______

[4] Let $\mathbf{v}_1 = (1, -1)$ and $\mathbf{v}_2 = (1, 1)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = 3\mathbf{v}_1 - \mathbf{v}_2, \quad L(\mathbf{v}_2) = 3\mathbf{v}_2 - \mathbf{v}_1.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$.

 Continued on (back of) page: ______

[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = -\mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y + z = 0$, and $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $W \subset \mathbb{R}^3$ defined by $x = y = 0$. Find a matrix that represents L with respect to the usual basis

$$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1).$$

 Continued on (back of) page: ______

[2] Let

$$\mathbf{v}_1 = (1, 1, 0, 1),\ \mathbf{v}_2 = (1, 0, -1, 0),\ \mathbf{v}_3 = (1, -3, 0, -1),\ \mathbf{v}_4 = (0, 1, -1, 0),\ \mathbf{v}_5 = (0, 1, 1, 1).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, and $\mathbf{v}_5$. Extend this basis to a basis for $\mathbb{R}^4$.

 Continued on (back of) page: ______

[4] Let $\mathbf{v}_1 = (1, 1)$ and $\mathbf{v}_2 = (1, 2)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) \;=\; \mathbf{v}_1 \;+\; 2\mathbf{v}_2, \quad L(\mathbf{v}_2) \;=\; \mathbf{v}_1 \;+\; \mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$.

Continued on (back of) page: ______

[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = 2\mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y + z = 0$, and $L(1,1,1) = (1,0,-1)$. Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1,0,0)$, $\mathbf{e}_2 = (0,1,0)$, $\mathbf{e}_3 = (0,0,1)$.

Continued on (back of) page: ______

[5] Let A be the 3×3 matrix determined by

$$A\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \qquad A\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \qquad A\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Find A.

 Continued on (back of) page: ______

[4] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace defined by $x - y + z = 0$, and $L(\mathbf{v}) = 0$ for all $\mathbf{v}$ belonging to the subspace defined by $x = y = 0$. Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0)$, $\mathbf{e}_3 = (0, 0, 1)$.

Continued on (back of) page: ______

[3] Find a 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \qquad A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$$

[4] Find a matrix representing the linear map from $\mathbb{R}^2$ to $\mathbb{R}^2$ which reflects first across the line $y = x$, then across the line $y = 2x$.

[5] Let $A : \mathbb{R}^2 \to \mathbb{R}^2$ be the matrix which flips the plane $\mathbb{R}^2$ across the line $3x = y$. Find A.

[4] Let

$$v_1 = (1,0,0), \qquad v_2 = (1,1,0), \qquad v_3 = (0,1,1)$$

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map such that

$$L(v_1) = v_2, \qquad L(v_2) = v_3, \qquad L(v_3) = v_1,$$

Find the matrix A (in standard coordinates) which represents the linear map L.

[4] Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects across the x-axis, and then reflects across the line $y = x$. Find a matrix A that represents L in standard coordinates.

[**4**] Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that $L(v) = v$ for all v on the line $x + 2y = 0$, and $L(v) = 2v$ for all v on the line $x = y$. Find a matrix A that represents L in standard coordinates.

Practice Problems 2

Linear Algebra, Dave Bayer, March 18, 2012

[1] Let V and W be the subspaces of $\mathbb{R}^2$ spanned by $(1,1)$ and $(1,2)$, respectively. Find vectors $v \in V$ and $w \in W$ so $v + w = (2,-1)$.

[2] Let V and W be the subspaces of $\mathbb{R}^2$ spanned by $(1,-1)$ and $(2,1)$, respectively. Find vectors $v \in V$ and $w \in W$ so $v + w = (1,1)$.

[3] Let V and W be the subspaces of $\mathbb{R}^2$ spanned by $(1,1)$ and $(1,4)$, respectively. Find vectors $v \in V$ and $w \in W$ so $v + w = (2,3)$.

[4] Let V be the subspace of $\mathbb{R}^3$ consisting of all solutions to the equation $x + y + z = 0$. Let W be the subspace of $\mathbb{R}^3$ spanned by $(1, 1, 0)$. Find vectors $v \in V$ and $w \in W$ so $v + w = (0, 0, 1)$.

[5] Let V be the subspace of $\mathbb{R}^3$ consisting of all solutions to the equation $x + y - z = 0$. Let W be the subspace of $\mathbb{R}^3$ spanned by $(1, 0, 4)$. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 1, 1)$.

[6] Let V be the subspace of $\mathbb{R}^3$ consisting of all solutions to the equation $x + 2y + z = 0$. Let W be the subspace of $\mathbb{R}^3$ spanned by $(1, 1, 1)$. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 1, 0)$.

[7] Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Let W be the orthogonal complement of V. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 0, 1, 0)$.

[8] Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Let W be the orthogonal complement of V. Find vectors $v \in V$ and $w \in W$ so $v + w = (0, 0, 1, 1)$.

[9] Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions to the system of equations

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Let W be the orthogonal complement of V. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 0, 0, 0)$.

[3] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 1)$, $(x_3, y_3) = (3, 1)$.

 Continued on page: ______

[4] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 2)$, $(x_3, y_3) = (2, 1)$.

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answer:

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work:

Continued on back of page ______

[3] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 2)$, $(x_3, y_3) = (2, 1)$, $(x_4, y_4) = (3, 0)$.

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answer:

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work:

Continued on back of page ______

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (-1, 0)$, $(x_2, y_2) = (0, 0)$, $(x_3, y_3) = (1, 1)$, $(x_4, y_4) = (2, 0)$.

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answer:

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work:

Continued on back of page ______

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$(x_1, y_1) = (-1, 0), \quad (x_2, y_2) = (0, 0), \quad (x_3, y_3) = (0, 2), \quad (x_4, y_4) = (1, 1).$$

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answer:

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work:

Continued on back of page ______

Exam 3

Linear Algebra, Dave Bayer, November 16, 2006

Name: ______________________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	[**5**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$(x_1, y_1) = (0, 0), \quad (x_2, y_2) = (1, 0), \quad (x_3, y_3) = (2, 1), \quad (x_4, y_4) = (3, 0).$$

 Continued on (back of) page: ______

Exam 3

Linear Algebra, Dave Bayer, April 17, 2007

Name: ____________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation. Please simplify each answer as far as possible.

[1] By least squares, find the equation of the form $z = ax + by + c$ which best fits the data

$$(x_1, y_1, z_1) = (0,0,0), \quad (x_2, y_2, z_2) = (1,0,0), \quad (x_3, y_3, z_3) = (0,1,0), \quad (x_4, y_4, z_4) = (1,1,1)$$

[2] By least squares, find the equation of the form $z = ax + by + c$ which best fits the data

$$(x_1, y_1, z_1) = (0, 0, 1), \quad (x_2, y_2, z_2) = (1, 0, 1), \quad (x_3, y_3, z_3) = (0, 1, 0), \quad (x_4, y_4, z_4) = (1, 1, 2)$$

Final Exam

Linear Algebra, Dave Bayer, May 10, 2011

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	[7] (5 pts)	[8] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} \quad = \quad \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$$

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 2 \end{bmatrix}$$

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & 3 \\ 2 & 0 \end{bmatrix}$$

[4] Find (s, t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

Continued on page: ______

[5] Find (s,t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

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answer:

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work:

Continued on back of page ______

[4] Find (s,t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \end{bmatrix}$.

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answer:

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work:

Continued on back of page ______

[**3**] Find (s,t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

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answer:

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work:

Continued on back of page ______

[3] Find (s,t) so $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$.

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answer:

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work:

Continued on back of page ______

[2] Find (s, t) so $\begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Continued on (back of) page: ______

[5] Find an orthogonal basis for the subspace $w + 2x + 3y + 4z = 0$ of $\mathbb{R}^4$.

Continued on page: ______

[3] Find an orthogonal basis for the subspace $w - x + y - z = 0$ of $\mathbb{R}^4$.

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answer:

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work:

Continued on back of page ______

Final Exam

Linear Algebra, Dave Bayer, May 13, 2003

Name: ______________________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^6$ consisting of all vectors (a, b, c, d, e, f) such that $a = b$, $c = d$, and $e = f$.

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answer:

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work:

Continued on back of page ______

[2] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors $(2, 1, 0, 0)$, $(0, 1, 1, 0)$, $(0, 0, 1, 2)$.

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answer:

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work:

Continued on back of page ______

Final Exam

Linear Algebra, Dave Bayer, May 11, 2004

Name: ______________________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors $(1,0,0,1)$, $(0,1,0,1)$, $(0,0,1,1)$.

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answer:

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work:

Continued on back of page ______

Final Exam

Linear Algebra, Dave Bayer, May 10, 2005

Name: ______________________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^5$ spanned by the vectors

$$(1, 0, -1, 0, 1), \quad (1, 0, 0, -1, 1), \quad (0, 1, -1, 0, 1), \quad (0, 1, 0, -1, 1).$$

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answer:

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work:

Continued on back of page ______

[3] Find an orthogonal basis for the subspace V of $\mathbb{R}^5$ spanned by the vectors

$$(1,0,-1,0,1), \quad (0,1,-1,1,0), \quad (1,-1,0,0,0), \quad (0,0,0,1,-1).$$

Continued on (back of) page: ______

[5] Define the inner product of two polynomials f and g by the rule

$$f \cdot g \quad = \quad \int_0^1 f(x)\, g(x)\, dx$$

Using this definition of the inner product, find an orthogonal basis for the vector space of all polynomials of degree ≤ 2.

 Continued on (back of) page: ______

Exam 4

Linear Algebra, Dave Bayer, May 10, 2007

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation. Please simplify each answer as far as possible.

[1] Find an orthogonal basis for the subspace V of $\mathbb{R}^5$ spanned by the vectors

$$(1, 0, -1, 0, 1) \quad (0, 1, -1, 0, 0) \quad (0, 0, 1, -1, 0)$$

[3] Define the inner product of two polynomials f and g by the rule

$$\langle f, g \rangle \quad = \quad \int_{-1}^{1} f(x)\, g(x)\, dx$$

Using this definition of the inner product, find an orthogonal basis for the vector space of all polynomials of degree $\leqslant 2$.

Exam 3 (Final)

Linear Algebra, Dave Bayer, December 20, 2007

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	[7] (5 pts)	[8] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the rows of the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

[5] Define the inner product of two polynomials f and g by the rule

$$\langle f, g \rangle \quad = \quad \int_0^1 f(x)\, g(1-x)\, dx$$

Using this definition of the inner product, find an orthogonal basis for the vector space of all polynomials of degree $\leqslant 2$.

[2] Extend the vector (1,1,1,2) to an orthogonal basis for $\mathbb{R}^4$.

[3] Let V be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$(1,1,1,2), \quad (2,1,1,1).$$

Find an orthogonal basis for the subspace V.

[3] Let V be the subspace of $\mathbb{R}^5$ spanned by the vectors

$$(1,1,1,0,0), \quad (0,0,1,1,1).$$

Find an orthogonal basis for the subspace V.

[**3**] Let V be the subspace of $\mathbb{R}^4$ spanned by the rows of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix}$$

Find the matrix A which projects $\mathbb{R}^4$ orthogonally onto the subspace V.

[3] Find the orthogonal projection of the vector (1,0,0,0) onto the subspace of $\mathbb{R}^4$ spanned by the vectors (1,1,1,0) and (0,1,1,1).

[4] Find the matrix A which projects $\mathbb{R}^4$ orthogonally onto the subspace spanned by the vectors (1,1,1,1) and (1,1,2,2).

[5] Let V be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$(1,1,1,1), \quad (1,1,2,2), \quad (1,1,3,3).$$

Find the matrix A which projects orthogonally onto the subspace V.

[5] Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Find the matrix A which projects orthogonally onto the subspace V.