Exam 3

Linear Algebra, Dave Bayer, 8:40 AM, April 16, 2013

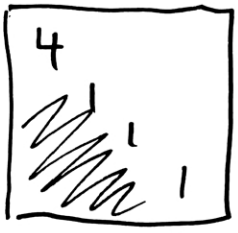
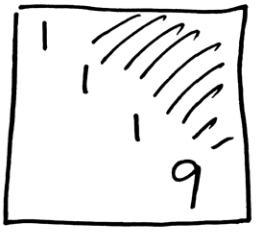
Name: _____ **Uni:** _____

[1]	[2]	[3]	[4]	[5]	**Total**

If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 4 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 9 & 1 \end{bmatrix}$$

$\det A = +1$  $+1$  $= 4+9 = 13$

(expand by minors)

[2] Find w/z where

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$w/z =$ Det $\begin{bmatrix} a & 2 & 3 & 4 \\ b & 1 & 2 & 3 \\ c & 0 & 1 & 2 \\ d & 0 & 0 & 1 \end{bmatrix}$ / Det $\begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 1 & 2 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & d \end{bmatrix}$ } d

$+a\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} - b\begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} + c\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix} - d\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

$= a - 2b + c$

⇓ row reduce

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ } 0

$$\boxed{w/z = \frac{a-2b+c}{d}}$$

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check $a=b=c=d=1$

$w/z = 0$ ✓

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ & 1 & 2 & 3 \\ & & 1 & 2 \\ & & & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (solve ⇑)

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check pick $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$ to find $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 17 \\ 11 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$w/z = 1/2 = \frac{17-2\cdot 11+6}{2}$ ✓

[3] Compute A^n for the matrix

$$A = \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$$

sum = trace = 2 $\quad\quad$ $\lambda = -2, 4$

prod = det = -8

$\lambda = -2$ $\quad A+2I: \begin{bmatrix} 1 & 1 \\ 5 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0$

$\lambda = 4$ $\quad A-4I: \begin{bmatrix} -5 & 1 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 5 \end{bmatrix} = 0$

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} -2 & \\ & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 1 & 1 \end{bmatrix}/6$$

$$A^n = (-2)^n \begin{bmatrix} 5 & -1 \\ -5 & 1 \end{bmatrix}/6 + 4^n \begin{bmatrix} 1 & 1 \\ 5 & 5 \end{bmatrix}/6$$

check $n=0$ I ✓

$n=1$ A $\begin{bmatrix} -10+4 & 2+4 \\ 10+20 & -2+20 \end{bmatrix}$ ✓

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

sum = trace = 4

prod = det $= 1\begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 \\ 2 & -1 \end{vmatrix} = 4 - 2(-2) + 2(-6) = -4$

$\lambda = -1, 1, 4$? try them.

$\lambda = -1$ $A+I$: $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = 0$ ✓

$\lambda = 1$ $A-I$: $\begin{bmatrix} 0 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = 0$ ✓

$\lambda = 4$ $A-4I$: $\begin{bmatrix} -3 & 2 & 2 \\ 2 & -2 & 2 \\ 2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 4 \\ +5 \\ 1 \end{bmatrix} = 0$ ✓

① + ② = [-1 0 4]

$-1,\ (1,0,-1)$
$1,\ (1,2,-2)$
$4,\ (4,5,1)$

check:

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 4 \\ 0 & 2 & 5 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 16 \\ 0 & 2 & 20 \\ 1 & -2 & 4 \end{bmatrix}$$

✓

[5] Compute A^n for the matrix

$$A = \begin{bmatrix} -1 & 2 & -4 \\ -1 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

trace = 4

det = 0

$\begin{vmatrix} -1 & 2 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & -4 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 0 & 3 \end{vmatrix} = 0 - 3 + 6 = 3$

$-|A - \lambda I| = \lambda^3 - 4\lambda^2 + 3\lambda = \lambda(\lambda-1)(\lambda-3) \quad \lambda = 0, 1, 3$

$\lambda = 0$, A: $\begin{bmatrix} -1 & 2 & 2 \\ -1 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 0$

$\lambda = 1$, A − I: $\begin{bmatrix} -2 & 2 & -4 \\ -1 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0$

① + 2② = [−6 0 0]

$\lambda = 3$, A − 3I: $\begin{bmatrix} -4 & 2 & -4 \\ -1 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ +1 \end{bmatrix} = 0$

	2	1	0	2	1
1	1	0	1	1	
0	2	1	0	2	
2	1	0	2	1	
1	1	0	1	1	

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & +1 \end{bmatrix} \begin{bmatrix} 0 & & \\ & 1 & \\ & & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -4 \\ 0 & 0 & 1 \end{bmatrix} / 1$$

$$A^n = 0^n \begin{bmatrix} 2 & -2 & 4 \\ 1 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} + 1^n \begin{bmatrix} -1 & 2 & -4 \\ -1 & 2 & -4 \\ 0 & 0 & 0 \end{bmatrix} + 3^n \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

need for check

$n = 0$: I ✓

$n = 1$: A $\begin{bmatrix} -1 & 2 & -4 \\ -1 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ ✓