Exam 3

Linear Algebra, Dave Bayer, 10:10 AM, April 16, 2013

Name: ______________________________ **Uni:** ______________

[1]	[2]	[3]	[4]	[5]	**Total**

If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] Compute the determinant of the matrix

$$A = \begin{bmatrix} 0 & 3 & 2 & 0 \\ 3 & 6 & 9 & 2 \\ 2 & 9 & 6 & 3 \\ 0 & 2 & 3 & 0 \end{bmatrix}$$

$$\det A = \det \begin{bmatrix} 0 & 3 & 2 & 0 \\ 3 & 0 & 0 & 2 \\ 2 & 0 & 0 & 3 \\ 0 & 2 & 3 & 0 \end{bmatrix} = \det \begin{bmatrix} 3 & 2 & & \\ 2 & 3 & & \\ & & 3 & 2 \\ & & 2 & 3 \end{bmatrix} = 5 \cdot 5 = \boxed{25}$$

(row reduce) (swap rows, cols)

[2] Find w/z where

$$\begin{bmatrix} a & b & c & d \\ 1 & 1 & 5 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$w/z = \underbrace{\det \begin{bmatrix} 1 & & & \\ 0 & 1 & 5 & 1 \\ 0 & & 1 & 1 \\ 0 & & & 2 \end{bmatrix}}_{2} \Big/ \underbrace{\underset{\text{triangular after col swaps}}{\det \begin{bmatrix} & & & 1 \\ 1 & 1 & 5 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix}}}_{-3}$$

$$\boxed{w/z = -2/3}$$

check $a, b, c, d = 1, 0, 0, 0$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 \\ 3 & 0 & 0 & 2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 5 & 1 & -1 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 2 & -3 \end{array}\right]$$

$w = 1$

$z = -3/2$

$$w/z = 1/{-3/2} = -2/3 \quad ✓$$

[3] Compute A^n for the matrix

$$A = \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix}$$

sum = trace = 1 $\quad \lambda = -3, 4$

prod = det = −12

$\lambda = -3 \quad A+3I : \begin{bmatrix} 2 & 2 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0$

$A-4I : \begin{bmatrix} -5 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = 0$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} -3 & \\ & 4 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ 1 & 1 \end{bmatrix} / 7$$

$$\boxed{A^n = (-3)^n \begin{bmatrix} 5 & -2 \\ -5 & 2 \end{bmatrix} / 7 + 4^n \begin{bmatrix} 2 & 2 \\ 5 & 5 \end{bmatrix} / 7}$$

check: $n=0 \quad I$ ✓

$n=1 \quad A \quad \begin{bmatrix} -15+8 & 6+8 \\ 15+20 & -6+20 \end{bmatrix}$ ✓

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

sum = trace = -1

prod = det $= 1\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix} + (-1)\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 8 - 4 = 4$

$\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} + \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = -5 + 1 = -4$

$\lambda = -2, -1, 2$? $-2(-1) + (-2)2 + (-1)2 = 2 - 4 - 2 = -4$ ✓

$\lambda = -2$ $A + 2I$: $\begin{bmatrix} 3 & 2 & -1 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ -5 \\ -1 \end{bmatrix} = 0$ ①+② = $[5\ 3\ 0]$

$\lambda = -1$ $A + I$: $\begin{bmatrix} 2 & 2 & -1 \\ 2 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} = 0$

$\lambda = 2$ $A - 2I$: $\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & 1 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$ ②−③ = $[0\ -4\ 4]$

$$\boxed{\begin{array}{ll} \lambda = -2 & (3, -5, -1) \\ \lambda = -1 & (1, -2, -2) \\ \lambda = 2 & (1, 1, 1) \end{array}}$$

check:

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 1 & 1 \\ -5 & -2 & 1 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -6 & -1 & 2 \\ 10 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$ ✓

[5] Compute A^n for the matrix

$$A = \begin{bmatrix} 4 & -4 & 4 \\ 2 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

trace $= 3$

det $= 0$

$$\begin{vmatrix} 4 & -4 \\ 2 & -2 \end{vmatrix} + \begin{vmatrix} 4 & 4 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} -2 & 3 \\ 0 & 1 \end{vmatrix} = 0 + 4 - 2 = 2$$

$$-|A - \lambda I| = \lambda^3 - 3\lambda^2 + 2\lambda = \lambda(\lambda-1)(\lambda-2) \quad \lambda = 0,1,2$$

$$\lambda = 0 \quad A: \begin{bmatrix} 4 & -4 & 4 \\ 2 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$\lambda = 1 \quad A - I: \begin{bmatrix} 3 & -4 & 4 \\ 2 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\lambda = 2 \quad A - 2I: \begin{bmatrix} 2 & -4 & 4 \\ 2 & -4 & 3 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & & \\ & 1 & \\ & & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 & -2 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \Big/ 1$$

$$\begin{array}{c|cccc} & 1 & 1 & 0 & 1 & 1 \\ \hline 0 & 1 & 1 & 0 & 1 \\ 2 & 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{array}$$

$$\boxed{A^n = 0^n \begin{bmatrix} \; \end{bmatrix} + 1^n \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} + 2^n \begin{bmatrix} 2 & -2 & 2 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}}$$

need for check

$$\begin{bmatrix} -1 & 2 & -2 \\ -1 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$n = 0$ I ✓

$$n = 1 \quad A \quad \begin{bmatrix} 4 & -4 & 4 \\ 2 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$