Exam 2  
Linear Algebra, Dave Bayer, 10:10 AM, March 12, 2013

Name: ________________________________  Uni: ________________

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If you need more than one page for a problem, clearly indicate on each page where to look next for your work.

[1] Find a basis for the set of solutions to the system of equations

\[
\begin{bmatrix} 1 & 1 & 1 & 2 & 0 \\ 2 & 2 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

Extend this basis to a basis for \( \mathbb{R}^5 \).
[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$(x_1, y_1) = (0, 1), \quad (x_2, y_2) = (1, 0), \quad (x_3, y_3) = (2, 2)$
[3] Let $L$ be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which projects orthogonally onto the subspace
\[ x + 2y + z = 0 \]
Find the matrix $A$ which represents $L$ in standard coordinates.
[4] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors

$$(1, 1, 1, 1), \quad (1, 2, 1, 2), \quad (2, 1, 2, 1), \quad (2, 2, 2, 2)$$
Let $V$ be the vector space of all polynomials of degree $\leqslant 3$ in the variable $x$ with coefficients in $\mathbb{R}$. Let $W$ be the subspace of polynomials satisfying $f(0) = f'(0) = 0$. Find an orthogonal basis for $W$ with respect to the inner product

$$\langle f, g \rangle = \int_{0}^{1} f(x)g(x) \, dx$$