Exam 2

Linear Algebra, Dave Bayer, 10:10 AM, March 12, 2013

Name: ______________________________ **Uni:** ______________

[1]	[2]	[3]	[4]	[5]	**Total**

If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] Find a basis for the set of solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 1 & 2 & 0 \\ 2 & 2 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Extend this basis to a basis for $\mathbb{R}^5$.

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$(x_1, y_1) = (0, 1), \quad (x_2, y_2) = (1, 0), \quad (x_3, y_3) = (2, 2)$$

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which projects orthogonally onto the subspace

$$x + 2y + z = 0$$

Find the matrix A which represents L in standard coordinates.

[4] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors

$$(1,1,1,1), \quad (1,2,1,2), \quad (2,1,2,1), \quad (2,2,2,2)$$

[5] Let V be the vector space of all polynomials of degree $\leqslant 3$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of polynomials satisfying $f(0) = f'(0) = 0$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)g(x)\,dx$$