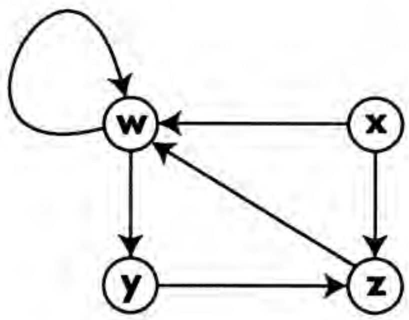
Exam 1 Alt

Linear Algebra, Dave Bayer, February 12, 2013

[1]	[2]	[3]	[4]	[5]	**Total**

If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

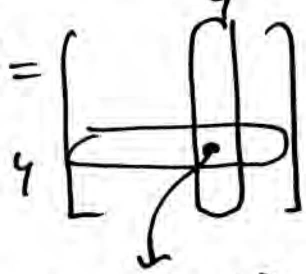
[1] Using matrix multiplication, count the number of paths of length nine from y to itself.



$$A = \text{out}\left\{\begin{array}{c|cccc} & \overbrace{w \;\; x \;\; y \;\; z}^{\text{in}} \\ w & 1 & 1 & 0 & 1 \\ x & 0 & 0 & 0 & 0 \\ y & 1 & 0 & 0 & 0 \\ z & 0 & 1 & 1 & 0 \end{array}\right.$$

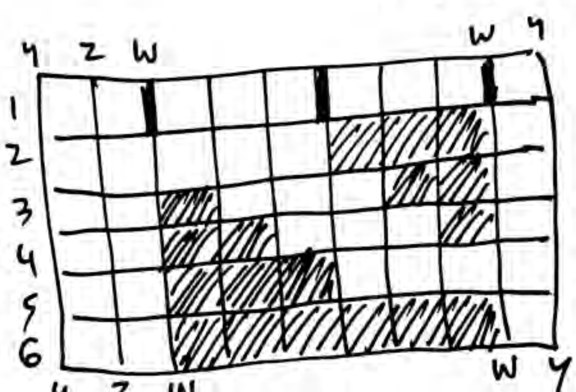
$$A^2 = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad A^3 = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} 6 & 7 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 4 & 5 & 2 & 3 \\ 3 & 3 & 1 & 2 \end{bmatrix} \qquad A^9 =$$



$$(1,2,1,1)\cdot(3,0,2,1) = 3+2+1 = 6$$

check:



← shade time at w

✓

$$\begin{array}{r} 1 \\ 4 \\ +\,1 \\ \hline 6 \end{array} \left.\begin{array}{l} \text{3 loops} \\ \text{2 loops} \\ \text{1 loop} \end{array}\right.$$

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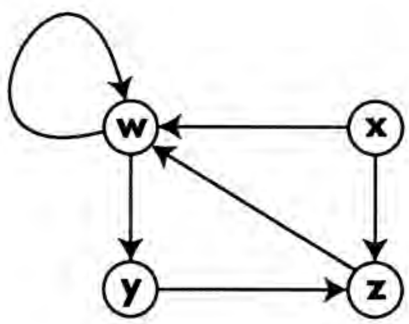

$$A = \text{out}\left\{\begin{array}{c|c} & \overbrace{\begin{matrix} w & x & y & z \end{matrix}}^{\text{in}} \\ \begin{matrix} w \\ x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}\right.$$

$$A^2 = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad A^3 = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

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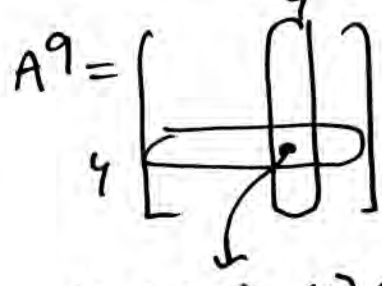
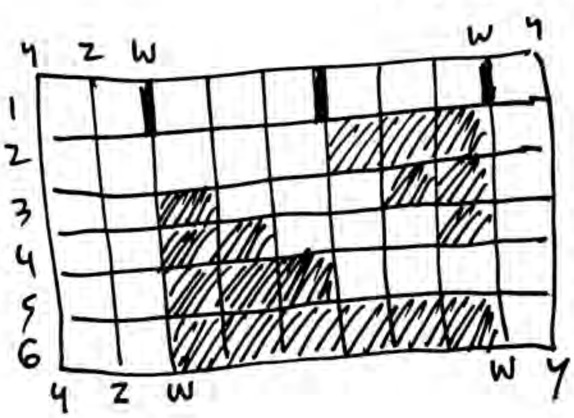


$(1,2,1,1)\cdot(3,0,2,1) = 3+2+1 = 6$

check:



← shade time at w

✓

$$\begin{array}{r|l} 1 & \text{3 loops} \\ 4 & \text{2 loops} \\ +\ 1 & \text{1 loop} \\ \hline 6 & \end{array}$$

[2] Solve the following system of equations.

$$\begin{bmatrix} 2 & 4 & 0 & 1 \\ 3 & 5 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 0 & 1 \\ 3 & 5 & 1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -3 & -5 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} \right) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

so $\boxed{\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -3 & -5 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}}$

[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \xRightarrow{\text{②} \leftarrow \text{②} - 2\text{①}} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \xRightarrow{\text{②} \leftarrow -\text{②}} \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \xRightarrow{\text{①} \leftarrow \text{①} - 3\text{②}} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \xRightarrow{\text{①} \leftarrow \text{①}/2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{A = \underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}}_{\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}}}_{\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}}}_{\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}}}$$

✓

[4] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

① find matrix using homog solution

$$\underbrace{\begin{bmatrix} -1 & 1 & 0 & -1 \\ -1 & 0 & 1 & -1 \end{bmatrix}}_{\text{fill in}} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} = 0$$

② find right hand side using particular solution

$$\begin{bmatrix} -1 & 1 & 0 & -1 \\ -1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \underbrace{\begin{bmatrix} -4 \\ -3 \end{bmatrix}}_{\text{fill in}}$$

$$\boxed{\begin{bmatrix} -1 & 1 & 0 & -1 \\ -1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \end{bmatrix}}$$

check $\begin{bmatrix} -1 & 1 & 0 & -1 \\ -1 & 0 & 1 & -1 \end{bmatrix} \left(\begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} \right) = \begin{bmatrix} -4 \\ -3 \end{bmatrix} + 0$ ✓

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & -1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right] \quad \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

check $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ also yields $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$\boxed{\text{point, } \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$