Solutions

Exam 1, February 12, 2013
10:10

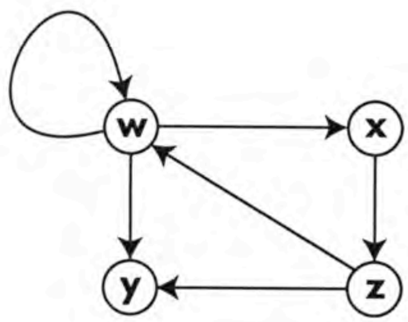
Exam 1

Linear Algebra, Section 003 (TR 10:10am – 11:25am), Dave Bayer, February 12, 2013

[1]	[2]	[3]	[4]	[5]	**Total**

If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] Using matrix multiplication, count the number of paths of length eight from z to itself.



(Your graph may vary, to discourage and detect copying.)

$$\text{out}\left\{\begin{array}{c|cccc|} & \multicolumn{4}{c}{\overbrace{\;w\quad x\quad y\quad z\;}^{\text{in}}} \\ w & 1 & 0 & 0 & 1 \\ x & 1 & 0 & 0 & 0 \\ y & 1 & 0 & 0 & 1 \\ z & 0 & 1 & 0 & 0 \end{array}\right. = A$$

$$A^2 = AA = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad A^4 = A^2A^2 = \begin{bmatrix} 3 & 1 & 0 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$A^8 = A^4A^4 = \begin{bmatrix} & \vdots \\ \cdots & 4 \end{bmatrix}$$

don't need other entries

$\boxed{\text{4 paths}}$

check:

z.w.w.w.w.w.w.x.z
z.w.x.z.w.w.w.x.z
z.w.w.x.z.w.w.x.z
z.w.w.w.x.z.w.x.z

10:10

[2] Solve the following system of equations.

$$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 6 & 8 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$②\leftarrow ② - ① \quad \left[\begin{array}{cccc|c} 2 & 3 & 1 & 0 & 2 \\ 4 & 5 & 0 & 1 & 3 \end{array}\right]$$

$$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & -3 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & -3 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}}$$

10:10

[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix} \xrightarrow{② \leftarrow ② - 3①} \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \xrightarrow{② \leftarrow -\frac{1}{4}②} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \xrightarrow{① \rightleftarrows ②} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\xleftarrow{② \leftarrow ② + 3①} \qquad \xleftarrow{② \leftarrow -4②} \qquad \xleftarrow{① \rightleftarrows ②}$$

$$A = \left(\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) I$$

$$\boxed{A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}}}$$

$$\begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix}$$

check ☑

(Be explicit what you consider the answer to be. Making us guess, or choose between alternatives, will cost points.)

10:10

[4] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ -4 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & 4 & -5 \end{bmatrix} \left(\begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ -4 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$\boxed{1}$ use homogeneous to fill in

particular

homogeneous

$\boxed{2}$ use particular to fill in

$$\boxed{\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & 4 & -5 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}}$$

10:10

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad \begin{aligned} 2s &= 2 \\ s &= 1 \end{aligned}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} t = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}}$$

check:

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} t \right) = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} t = \begin{bmatrix} 0 \\ 2 \end{bmatrix} ✓$$