[1] Let $A$ be the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ -1 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 1 & -2 & 1 \end{bmatrix}.$$ 

Compute the row space and column space of $A$. 
Let
\[ \mathbf{v}_1 = (1, 1, 0, 0), \mathbf{v}_2 = (1, 0, 1, 0), \mathbf{v}_3 = (1, 0, 0, -1), \mathbf{v}_4 = (0, 1, -1, 0), \mathbf{v}_5 = (0, 1, 0, 1). \]

Find a basis for the subspace \( V \subset \mathbb{R}^4 \) spanned by \( \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \) and \( \mathbf{v}_5 \). Extend this basis to a basis for \( \mathbb{R}^4 \).
Let $V$ be the vector space of all polynomials $f(x)$ of degree $\leq 3$. Find a basis for the subspace $W$ defined by

$$f(-1) = f(0), \quad f(-1) = f(1), \quad f(0) = f(1).$$

Extend this basis to a basis for $V$. 
Let \( v_1 = (1, -1) \) and \( v_2 = (1, 1) \). Let \( L : \mathbb{R}^2 \to \mathbb{R}^2 \) be the linear transformation such that

\[
L(v_1) = 3v_1 - v_2, \quad L(v_2) = 3v_2 - v_1.
\]

Find a matrix that represents \( L \) with respect to the usual basis \( e_1 = (1, 0), \ e_2 = (0, 1) \).
Let \( L : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) be the linear transformation such that \( L(v) = -v \) for all \( v \) belonging to the subspace \( V \subset \mathbb{R}^3 \) defined by \( x + y + z = 0 \), and \( L(v) = v \) for all \( v \) belonging to the subspace \( W \subset \mathbb{R}^3 \) defined by \( x = y = 0 \). Find a matrix that represents \( L \) with respect to the usual basis 
\[
e_1 = (1, 0, 0), \quad e_2 = (0, 1, 0), \quad e_3 = (0, 0, 1).
\]
Problem: _____