Exam 1

Linear Algebra, Dave Bayer, February 17, 2004

Name: _____

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[1] What is the set of all solutions to the following system of equations?

$$
\begin{bmatrix}
1 & -1 & 1 & 1 \\
-1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 \\
-1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
w \\ x \\ y \\ z
\end{bmatrix}
=
\begin{bmatrix}
1 \\ 1 \\ 1 \\ 1
\end{bmatrix}
$$

[2] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

[3] What is the determinant of the following 4×4 matrix?

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

[**4**] Using Cramer's rule, find w satisfying the following system of equations:

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

[5] Give a formula for the matrix which is inverse to:

$$\begin{bmatrix} a & 1 & 0 & 0 \\ 0 & b & 1 & 0 \\ 0 & 0 & c & 1 \\ 0 & 0 & 0 & d \end{bmatrix}$$

The inverse is:

$$\begin{bmatrix} \dfrac{1}{a} & -\dfrac{1}{ab} & \dfrac{1}{abc} & -\dfrac{1}{abcd} \\[2mm] 0 & \dfrac{1}{b} & -\dfrac{1}{bc} & \dfrac{1}{bcd} \\[2mm] 0 & 0 & \dfrac{1}{c} & -\dfrac{1}{cd} \\[2mm] 0 & 0 & 0 & \dfrac{1}{d} \end{bmatrix}$$

[6] What is the determinant of the following 8×8 matrix?

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$