Linear Algebra, Dave Bayer

Our final exam will consist of eight questions. Three questions will be review, and five questions will be on new material.

The following are the new skills that one needs to learn for this exam. In each case the corresponding matrix may be a 2×2 matrix or a 3×3 matrix. The eigenvalues will be integers; they might not be distinct.

- Find the characteristic equation and a system of eigenvalues and eigenvectors of a matrix.
- Find a formula for the nth power of a matrix.
- Find a formula for the matrix exponential of a matrix.
- Solve a linear system of differential equations, with an initial condition.
- Express a quadratic form as a sum of squares of orthogonal linear forms.
- Solve a recurrence relation in closed form.

This material is covered in chapters eight and nine of Bretscher, and in past exam problems. You are encouraged to read the chapters in Bretscher carefully.

Homework will count as 10% of your course grade. The homework for our Final Exam is given below. All homework must be submitted on or before the last day of classes, December 12. There is a homework box on the fourth floor of the Mathematics building for your section of Linear Algebra. Please turn in homework to the box corresponding to your section. Please write your uni very clearly on each page of homework.

Please hand in the following problems; they are the same in both the 5e and 4e editions of Bretscher. (You are encouraged to work similar problems for your own practice.)

- 8.1 [4], 8.2 [4]
- 9.1 [12], 9.3 [14]

What follows on the remaining pages of this study guide are practice problems for our final exam, taken from past semesters of the course.

In addition to these problems, a comprehensive set of review problems can be found in the problem set

- (Practice4-F13-LinearAlgebra)
- (Practice4-F13-HandSolus-LinearAlgebra)
- (Practice4-F13-SolutionKey-LinearAlgebra)

The sources for the following problems, along with many solutions, can be found on our Linear Algebra Course Materials page:

https://www.math.columbia.edu/~bayer/LinearAlgebra/

(F16 8:40 Exam 2) (Solutions)

[4] Find a system of eigenvalues and eigenvectors for the matrix A, and find a formula for A^n, where

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix}$$

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad \begin{bmatrix} 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 0 & 0 \\ 2 & 1 & -1 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find a formula for $f(n)$.

(F16 10:10 Exam 2) (Solutions)

[4] Find a system of eigenvalues and eigenvectors for the matrix A, and find a formula for A^n, where

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}$$

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad \begin{bmatrix} 4 \end{bmatrix} \qquad \begin{bmatrix} 4 & 1 \\ 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 4 & 1 & 0 \\ 3 & 4 & 1 \\ 0 & 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ 3 & 4 & 1 & 0 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 3 & 4 \end{bmatrix}$$

Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find a formula for $f(n)$.

(F16 8:40 Final) (Solutions)

[2] Find the inverse to the matrix

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -1 & -2 & -2 \\ 1 & 3 & 3 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[8] Express the quadratic form

$$2x^2 - 2xy + 3y^2 + 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

(F16 10:10 Final) (Solutions)

[2] Find the inverse to the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} -2 & 2 \\ -2 & 3 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 5 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 2 \\ 1 & -2 & -1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

[8] Express the quadratic form

$$x^2 - 2xy + 2y^2 + 2xz + 2z^2$$

as a sum of squares of orthogonal linear forms.

(F15 Homework 3)

[6] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 0 \\ 2 & -1 & -1 \end{bmatrix}$$

(F15 Exam 3)

[5] Find a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

(F15 Homework 4)

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$$

[3] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & -3 \\ 1 & -3 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 5 & -4 \\ 1 & 1 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 2 \\ -1 & -2 & 3 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[9] Express the quadratic form

$$3x^2 + 3y^2 + 2xz + 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

(F15 Final) (Solutions)

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 0 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[7] Express the quadratic form

$$2x^2 + 2y^2 - 2xz + 2yz + 3z^2$$

as a sum of squares of othogonal linear forms.

[8] Solve for z in the system of differential equations

$$\begin{aligned} y'' &= 2y' + y + z \\ z' &= -2y' + 2y + z \end{aligned}$$

where

$$y(0) = y'(0) = 0, \qquad z(0) = 1$$

(F14 Practice 3) (Solutions)

[6] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

(F14 Homework 3) (Solutions)

[6] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 0 \\ 2 & -1 & -1 \end{bmatrix}$$

(F14 Practice 4) (Solutions)

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 0 \end{bmatrix}$$

[3] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 3 & 3 & 1 \\ -1 & 2 & -2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[9] Express the quadratic form

$$3x^2 + 3y^2 - 2xz + 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

(F14 Homework 4) (Solutions)

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$$

[3] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & -3 \\ 1 & -3 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 5 & -4 \\ 1 & 1 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 2 \\ -1 & -2 & 3 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[9] Express the quadratic form

$$3x^2 + 3y^2 + 2xz + 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

(F14 8:40 Final) (Solutions)

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 2 \\ -2 & -2 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[8] Express the quadratic form

$$2x^2 - 2xy + 3y^2 + 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

(F14 11:40 Final) (Solutions)

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

[8] Express the quadratic form

$$2x^2 + 2xy + 3y^2 - 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

(S14 Exam 3) (Solutions)

[4] Find A^n where A is the matrix

$$\begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix}$$

Your final answer should be in the form

$$A^n = r^n B + s^n C$$

[5] Find $f(n)$, where $f(n)$ is the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 & 0 \\ 3 & 5 & 2 \\ 0 & 3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 & 0 & 0 \\ 3 & 5 & 2 & 0 \\ 0 & 3 & 5 & 2 \\ 0 & 0 & 3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 & 0 & 0 & 0 \\ 3 & 5 & 2 & 0 & 0 \\ 0 & 3 & 5 & 2 & 0 \\ 0 & 0 & 3 & 5 & 2 \\ 0 & 0 & 0 & 3 & 5 \end{bmatrix}$$

Your final answer should be in the form $f(n) = a\, r^n + b\, s^n$

(S14 Final) (Solutions)

[3] Find $f(n)$, where $f(n)$ is the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 \\ 1 & 3 & 2 \\ 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Your final answer should be in the form $f(n) = a\, r^n + b\, s^n$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -2 & 2 & -1 \\ -1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

[8] Express the quadratic form

$$3x^2 + 2xy + 2y^2 + 2yz + 3z^2$$

as a sum of squares of othogonal linear forms.

(F13 Exam 3) (Solutions)

[5] Find A^n where A is the matrix

$$\begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}$$

(F13 Final) (Solutions)

[4] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

[5] Express the quadratic form

$$-4xy + 3y^2$$

as a sum of squares of othogonal linear forms.

[6] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = 3\,f(n-1) \;-\; 2\,f(n-2)$$

[7] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} -2 & 2 & -1 \\ -1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

(S13 8:40 Exam 3) (Solutions)

[3] Compute A^n for the matrix

$$A \;=\; \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$$

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A \;=\; \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

[5] Compute A^n for the matrix

$$A \;=\; \begin{bmatrix} -1 & 2 & -4 \\ -1 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

(S13 10:10 Exam 3) (Solutions)

[3] Compute A^n for the matrix

$$A \;=\; \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix}$$

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A \;=\; \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

[5] Compute A^n for the matrix

$$A = \begin{bmatrix} 4 & -4 & 4 \\ 2 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

(S13 Alt Exam 3) (Solutions)

[3] Compute A^n for the matrix

$$A = \begin{bmatrix} -1 & 3 \\ 5 & 1 \end{bmatrix}$$

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$$

[5] Compute A^n for the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -3 & 3 \\ 1 & -6 & 6 \end{bmatrix}$$

(S13 8:40 Final)

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 2 \\ -2 & -3 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 5 & 1 \\ -4 & 1 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & -1 & 2 \\ 2 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

[8] Express

$$3x^2 + 3y^2 + 3z^2 - 2xy + 2xz + 2yz$$

as a linear combination of squares of orthogonal linear forms.

(S13 10:10 Final)

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 4 & 1 \\ -4 & 0 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -4 & 0 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -1 & 1 & 1 \\ -2 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

[8] Express

$$x^2 + y^2 + z^2 - 2xy + 2xz + 2yz$$

as a linear combination of squares of orthogonal linear forms.

(S13 Alt Final)

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & -3 \\ -1 & 0 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

[8] Express

$$3x^2 + 3y^2 + 3z^2 - 2xy + 2xz - 2yz$$

as a linear combination of squares of orthogonal linear forms.

(S12 Practice Final A) (Solutions)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} -1 & 2 \\ 3 & -2 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$$

[5] Express $x^2 + 6xy + y^2$ as a linear combination of squares of orthogonal linear forms.

[6] Convert the differential equation $y'' - 3y' + 2y = 0$ to matrix form, and solve by exponentiating.

[7] Find e^{At} where A is the matrix

$$\begin{bmatrix} 1 & -4 & 2 \\ 0 & 2 & 0 \\ 0 & -2 & 3 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ -2 & 4 & 1 \\ 2 & -2 & 0 \end{bmatrix}$$

(S12 Practice Final B)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$\begin{bmatrix} -1 & -1 \\ 3 & 3 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$$

[5] Express $-3x^2 + 8xy + 3y^2$ as a linear combination of squares of orthogonal linear forms.

[6] Convert the differential equation $y'' - 5y' + 6y = 0$ to matrix form, and solve by exponentiating.

[7] Find e^{At} where A is the matrix

$$\begin{bmatrix} 4 & -2 & 1 \\ -2 & 4 & -2 \\ -6 & 6 & -3 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(S12 Practice Final C)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$$

[5] Express $-x^2 + 6xy - y^2$ as a linear combination of squares of orthogonal linear forms.

[6] Convert the differential equation $y'' - 2y' + y = 0$ to matrix form, and solve by exponentiating.

[7] Find e^{At} where A is the matrix

$$\begin{bmatrix} -2 & 2 & -2 \\ -4 & 4 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$\begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 3 \end{bmatrix}$$

(S12 Practice Final D)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$\begin{bmatrix} -1 & -2 \\ 2 & 4 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$\begin{bmatrix} -3 & -1 \\ 4 & 1 \end{bmatrix}$$

[5] Express $-8x^2 + 12xy + 8y^2$ as a linear combination of squares of orthogonal linear forms.

[6] Convert the differential equation $y'' - 2y' - 3y = 0$ to matrix form, and solve by exponentiating.

[7] Find e^{At} where A is the matrix

$$\begin{bmatrix} 4 & -3 & -1 \\ 3 & -2 & -1 \\ 2 & -2 & 0 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$\begin{bmatrix} -3 & 4 & -2 \\ -3 & 4 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$

(S12 9:10 Final)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$\begin{bmatrix} 4 & -1 \\ 1 & 6 \end{bmatrix}$$

[5] Express $2x^2 - 8xy + 2y^2$ as a linear combination of squares of orthogonal linear forms.

[6] Convert the differential equation $y'' - 4y' + 4y = 0$ to matrix form, and solve by exponentiating.

[7] Find e^{At} where A is the matrix

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 6 & -8 \\ 0 & 4 & -6 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$\begin{bmatrix} 3 & -4 & 2 \\ 4 & -5 & 3 \\ 4 & -4 & 3 \end{bmatrix}$$

(S12 11:00 Final)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} 3 & 3 \\ 4 & -1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$\begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$\begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}$$

[5] Express $9x^2 + 4xy + 6y^2$ as a linear combination of squares of orthogonal linear forms.

[6] Convert the differential equation $y'' - 4y' + 3y = 0$ to matrix form, and solve by exponentiating.

[7] Find e^{At} where A is the matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$\begin{bmatrix} 0 & 4 & -4 \\ 2 & -2 & 1 \\ 3 & -6 & 5 \end{bmatrix}$$

(S11 Final) (Solutions)

[6] Find the matrix exponential e^{At}, for the matrix

$$A = \begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}$$

[7] Find the matrix exponential e^{At}, for the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

[8] Find a formula for A^n, for the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 3 \end{bmatrix}$$