F16 10:10 Final Exam Problem 1
Linear Algebra, Dave Bayer

test1b3p1

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Test 1

Name Solutions Uni

[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} s \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} t$$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}}$$

F16 10:10 Final Exam Problem 2
Linear Algebra, Dave Bayer

test1b3p2

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Test 1

[2] Find the inverse to the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{+4} \begin{bmatrix} -4 & 0 & 4 \\ -2 & 3 & 1 \\ 4 & -2 & -2 \end{bmatrix}$$

For full credit you <u>must</u> leave a positive denominator.

F16 10:10 Final Exam Problem 3
Linear Algebra, Dave Bayer

test1b3p3

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Test 1

[3] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} -2 & 2 \\ -2 & 3 \end{bmatrix}$$

$$\boxed{A^n \;=\; \frac{\square}{\square}\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}}$$

$$\lambda = -1, 2 \qquad A^n \;=\; \frac{(-1)^n}{3}\begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix} + \frac{2^n}{3}\begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix}$$

F16 10:10 Final Exam Problem 4
Linear Algebra, Dave Bayer

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test1b3p4

Test 1

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 5 \end{bmatrix}$$

$$\boxed{A^n = \frac{\square}{\square}\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}}$$

$$\lambda = 4, 4 \qquad A^n = 4^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n\,4^{n-1} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

F16 10:10 Final Exam Problem 5
Linear Algebra, Dave Bayer

test1b3p5

[Reserved for Score]

Test 1

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

$$A^n = \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$\lambda = 1, 2, 4 \qquad A^n = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{2^n}{2}\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix} + \frac{4^n}{2}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

F16 10:10 Final Exam Problem 6
Linear Algebra, Dave Bayer

test1b3p6

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Test 1

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$e^{At}3 = \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$\lambda = 0, 2, 2 \qquad e^{At} = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} + te^{2t}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Test 1

[7] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 2 \\ 1 & -2 & -1 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\boxed{\; y \;=\; \frac{\square}{\square}\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \;}$$

$$\lambda = 1, 1, 1$$

$$e^{At} \;=\; e^t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^t \begin{bmatrix} 0 & -2 & -2 \\ -1 & 2 & 2 \\ 1 & -2 & -2 \end{bmatrix} + \frac{t^2e^t}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & -2 & -2 \end{bmatrix}$$

$$y \;=\; e^t \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + te^t \begin{bmatrix} -6 \\ 6 \\ -6 \end{bmatrix} + \frac{t^2e^t}{2} \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix}$$

F16 10:10 Final Exam Problem 8
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test1b3p8

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Test 1

[8] Express the quadratic form

$$x^2 \;-\; 2xy \;+\; 2y^2 \;+\; 2xz \;+\; 2z^2$$

as a sum of squares of orthogonal linear forms.

$$\boxed{\;\square\left(\;\square\;\right)^2 + \square\left(\;\square\;\right)^2 + \square\left(\;\square\;\right)^2\;}$$

$$\lambda = 0, 2, 3 \qquad A \;=\; \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \;=\; \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \;+\; \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$(y \;+\; z)^2 \;+\; (x \;-\; y \;+\; z)^2$$