F14 Practice 2
Linear Algebra, Dave Bayer

[1] Find the 2×2 matrix which reflects across the line $x - 2y = 0$.

[2] Find the 3×3 matrix which vanishes on the plane $x + y + z = 0$, and maps the vector $(1, 0, 0)$ to itself.

[3] Find the 3×3 matrix which vanishes on the vector $(1, 1, 1)$, and maps each point on the plane $x + y = 0$ to itself.

[4] Find the 3×3 matrix that projects orthogonally onto the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} t$$

[5] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + y + z = 0$$

[6] Find the row space and the column space of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 2 & 3 & 3 & 3 & 3 \end{bmatrix}$$

[7] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 2 \end{bmatrix}$$

[8] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1, -1, 0, 0) \qquad (0, 1, -1, 0) \qquad (0, 0, 1, -1) \qquad (1, 0, -1, 0) \qquad (0, 1, 0, -1)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[9] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace consisting of those polynomials $f(x)$ such that $f(1) = 0$. Find the orthogonal projection of the polynomial x onto the subspace W, with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx$$