F14 Homework 3
Linear Algebra, Dave Bayer

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 1 & 3 & 3 & 1 \\ 1 & 4 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 \\ 1 & 3 & 1 & 2 & 1 \\ 5 & 1 & 4 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

[4] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 3 & a & 2 \\ 2 & b & 3 \\ 1 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[5] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$$

[6] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 0 \\ 2 & -1 & -1 \end{bmatrix}$$

[7] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = -1, \quad f(1) = 2$$
$$f(n) = f(n-1) + f(n-2)$$

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[8] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = 1, \quad f(1) = 1, \quad g(1) = 1$$
$$f(n) \;=\; f(n-1) \;+\; g(n-1)$$
$$g(n) \;=\; f(n-1) \;+\; f(n-2)$$

[9] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad \begin{bmatrix} 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.