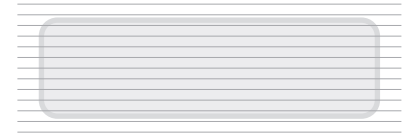


Exam 01

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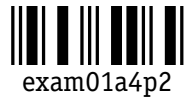


[1] Find the intersection of the following two affine subspaces of \mathbb{R}^4 .

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

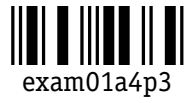
$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} + \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} t$$



Exam 01

[2] Find the 3×3 matrix A that maps the vector $(1, 2, 1)$ to $(3, 6, 3)$, and maps each point on the plane $x + y + z = 0$ to the zero vector.

$$A = \frac{1}{\boxed{}} \begin{bmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{bmatrix}$$

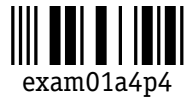


Exam 01

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

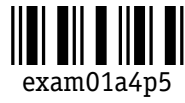


Exam 01

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 2 \\ -2 & -2 \end{bmatrix}$$

$$A^n = \frac{\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}^n}{\square} \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + \frac{\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}^n}{\square} \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

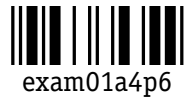


Exam 01

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y = \frac{\begin{bmatrix} \\ \end{bmatrix}}{\begin{bmatrix} \\ \end{bmatrix}} \begin{bmatrix} \\ \end{bmatrix} + \frac{\begin{bmatrix} \\ \end{bmatrix}}{\begin{bmatrix} \\ \end{bmatrix}} \begin{bmatrix} \\ \end{bmatrix}$$

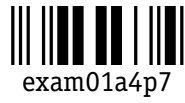


Exam 01

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$e^{At} = \frac{\begin{matrix} \square \\ \square \end{matrix}}{\begin{matrix} \square \\ \square \end{matrix}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\begin{matrix} \square \\ \square \end{matrix}}{\begin{matrix} \square \\ \square \end{matrix}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\begin{matrix} \square \\ \square \end{matrix}}{\begin{matrix} \square \\ \square \end{matrix}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

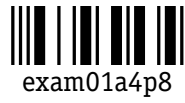


Exam 01

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$y = \frac{\begin{bmatrix} \\ \\ \end{bmatrix}}{\begin{bmatrix} \\ \end{bmatrix}} + \frac{\begin{bmatrix} \\ \\ \end{bmatrix}}{\begin{bmatrix} \\ \end{bmatrix}} + \frac{\begin{bmatrix} \\ \\ \end{bmatrix}}{\begin{bmatrix} \\ \end{bmatrix}}$$



Exam 01

[8] Express the quadratic form

$$2x^2 - 2xy + 3y^2 + 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

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