Exam 2
Linear Algebra, Dave Bayer, October 22, 2013

[1] Find a basis for the subspace $V$ of $\mathbb{R}^4$ spanned by the vectors

$\begin{align*}
(2, 0, 1, 0), & \quad (2, 0, 0, 1), \quad (0, 2, 1, 0), \quad (0, 2, 0, 1)
\end{align*}$

Extend this basis to a basis for $\mathbb{R}^4$.

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$\begin{align*}
(x_1, y_1) &= (−1, 0), & (x_2, y_2) &= (0, 0), & (x_3, y_3) &= (1, 0), & (x_4, y_4) &= (2, 1)
\end{align*}$

[3] Let $L$ be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which projects orthogonally onto the subspace $V$ spanned by $(1, −1, 0)$ and $(0, 2, 1)$. Find the matrix $A$ which represents $L$ in standard coordinates.

[4] Let $V$ be the vector space of all polynomials of degree $\leq 2$ in the variable $x$ with coefficients in $\mathbb{R}$. Let $W$ be the subspace of polynomials satisfying $f(2) = 0$. Find an orthogonal basis for $W$ with respect to the inner product

$\langle f, g \rangle = \int_0^1 f(x)g(x) \, dx$

[5] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ defined by the equation $w + x - 2y - 2z = 0$. Extend this basis to a orthogonal basis for $\mathbb{R}^4$. 