Exam 3 (Final)

Linear Algebra, Dave Bayer, December 20, 2007

Name: _____

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	[7] (5 pts)	[8] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the rows of the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

[2] By least squares, find the equation of the form $z = ax + by + c$ which best fits the data

$$(x_1, y_1, z_1) = (0, 0, 1), \quad (x_2, y_2, z_2) = (1, 0, 1), \quad (x_3, y_3, z_3) = (0, 1, 0), \quad (x_4, y_4, z_4) = (1, 1, 2)$$

[**3**] Let V be the subspace of $\mathbb{R}^4$ spanned by the rows of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix}$$

Find the matrix A which projects $\mathbb{R}^4$ orthogonally onto the subspace V.

[**4**] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 3$. Find a basis for the subspace W defined by

$$f(0) = f(1) = f(2)$$

Extend this basis to a basis for V.

[5] Define the inner product of two polynomials f and g by the rule

$$\langle f, g \rangle \quad = \quad \int_0^1 f(x)\, g(1-x)\, dx$$

Using this definition of the inner product, find an orthogonal basis for the vector space of all polynomials of degree $\leqslant 2$.

[**6**] Express the following quadratic form as a linear combination of squares of orthogonal linear forms:

$$3x^2 + 4xy + 6y^2$$

[7] Express the following quadratic form as a linear combination of squares of orthogonal linear forms:

$$2xy + 4xz + 4yz + 3z^2$$

[**8**] Find e^{At} for the matrix

$$A = \begin{bmatrix} 4 & 2 & 1 \\ -2 & -1 & -1 \\ -8 & -4 & -1 \end{bmatrix}$$

[9]