Exam 3

Linear Algebra, Dave Bayer, November 16, 2006

Name: ______________________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	[**5**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$(x_1, y_1) = (0, 0), \quad (x_2, y_2) = (1, 0), \quad (x_3, y_3) = (2, 1), \quad (x_4, y_4) = (3, 0).$$

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[2] Find (s,t) so $\begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

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[3] Find an orthogonal basis for the subspace V of $\mathbb{R}^5$ spanned by the vectors

$$(1, 0, -1, 0, 1), \quad (0, 1, -1, 1, 0), \quad (1, -1, 0, 0, 0), \quad (0, 0, 0, 1, -1).$$

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[**4**] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace defined by $x - y + z = 0$, and $L(\mathbf{v}) = 0$ for all $\mathbf{v}$ belonging to the subspace defined by $x = y = 0$. Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0)$, $\mathbf{e}_3 = (0, 0, 1)$.

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[5] Define the inner product of two polynomials f and g by the rule

$$f \cdot g \quad = \quad \int_0^1 f(x)\, g(x)\, dx$$

Using this definition of the inner product, find an orthogonal basis for the vector space of all polynomials of degree ≤ 2.

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