Second midterm

Dave Bayer, Modern Algebra, November 18, 1998

[1] Find the multiplicative inverse of 23 mod 103.

[2] *Prove* **ONE** *of the following two assertions:*

(a) Let V and W be subspaces of a vector space U. Then

$$\dim(V) + \dim(W) \quad = \quad \dim(V \cap W) + \dim(V + W).$$

(b) Let $T : V \to W$ be a linear transformation of vector spaces. Then

$$\dim(\ker(T)) + \dim(\mathrm{image}(T)) \quad = \quad \dim(V).$$

[3] Let

$$A = \begin{bmatrix} 5 & -4 \\ 1 & 1 \end{bmatrix}.$$

Find a change of basis matrix B so $A = B\,C\,B^{-1}$ where C is in Jordan canonical form. Use B and C to give an expression for e^{At}. You do not need to multiply this expression out.

[4] Let $T : V \to V$ be a linear transformation from the n-dimensional vector space V to itself, such that $T^2 = 0$. Prove that for some basis $\mathbf{v}_1, \ldots, \mathbf{v}_n$ of V, the matrix A for T with respect to this basis is in Jordan canonical form.

[5] Let

$$A = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}.$$

Find a formula for e^{At}.