[1] Find a pair of inverse ring isomorphisms between $\mathbb{Z}/91\mathbb{Z}$ and $\mathbb{Z}/7\mathbb{Z} \times \mathbb{Z}/13\mathbb{Z}$. Show that your maps are in fact inverse to each other. Using these maps, compute $5^{26}$ mod 91.
[2] Find a pair of inverse ring isomorphisms between \( \mathbb{Z}/187\mathbb{Z} \) and \( \mathbb{Z}/11\mathbb{Z} \times \mathbb{Z}/17\mathbb{Z} \). Show that your maps are in fact inverse to each other. Using these maps, compute \( 3^{32} \mod 187 \).
[3] Find a pair of inverse ring isomorphisms between \( \mathbb{Z}/144\mathbb{Z} \) and \( \mathbb{Z}/9\mathbb{Z} \times \mathbb{Z}/16\mathbb{Z} \). Show that your maps are in fact inverse to each other. Using these maps, compute \( 5^{25} \mod 144 \).
A message is represented as an integer $a \mod 55$. You receive the encrypted message $a^7 \equiv 13 \mod 55$. What is $a$?
A message is represented as an integer \( \alpha \mod 91 \). You receive the encrypted message \( \alpha^{17} \equiv 61 \mod 91 \). What is \( \alpha \)?
A message is represented as an integer $a \mod 187$. You receive the encrypted message $a^9 \equiv 60 \mod 187$. What is $a$?
[7] Let $A$ be an $n \times n$ matrix with entries in $\mathbb{R}$, satisfying the polynomial relation

$$(x - 2)(x - 3) = 0$$

Find a formula for $e^{At}$ as a polynomial expression in $A$. Give an example of a matrix $A$ for which this is the minimal polynomial relation, and check your formula using this matrix.
Let $A$ be an $n \times n$ matrix with entries in $\mathbb{R}$, satisfying the polynomial relation

$$(x - 2)^2 = 0$$

Find a formula for $e^{At}$ as a polynomial expression in $A$. Give an example of a matrix $A$ for which this is the minimal polynomial relation, and check your formula using this matrix.
[9] Let $A$ be an $n \times n$ matrix with entries in $\mathbb{R}$, satisfying the polynomial relation

$$(x - 2)^2(x - 3) = 0$$

Find a formula for $e^{At}$ as a polynomial expression in $A$. Give an example of a matrix $A$ for which this is the minimal polynomial relation, and check your formula using this matrix.
[10] Construct the finite field $\mathbb{F}_4$ as an extension of $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$, by finding an irreducible polynomial of degree 2 with coefficients in $\mathbb{F}_2$. What are the two roots of your irreducible polynomial?
[11] Construct the finite field $\mathbb{F}_8$ as an extension of $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$, by finding an irreducible polynomial of degree 3 with coefficients in $\mathbb{F}_2$. What are the three roots of your irreducible polynomial?
Construct the finite field \( \mathbb{F}_9 \) as an extension of \( \mathbb{F}_3 = \mathbb{Z}/3\mathbb{Z} \), by finding an irreducible polynomial of degree 2 with coefficients in \( \mathbb{F}_3 \). What are the two roots of your irreducible polynomial?
[13] Construct the finite field $\mathbb{F}_{16}$ as an extension of $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$, by finding an irreducible polynomial of degree 4 with coefficients in $\mathbb{F}_2$. What are the four roots of your irreducible polynomial?
[14] Construct the finite field $\mathbb{F}_{25}$ as an extension of $\mathbb{F}_5 = \mathbb{Z}/5\mathbb{Z}$, by finding an irreducible polynomial of degree 2 with coefficients in $\mathbb{F}_5$. What are the two roots of your irreducible polynomial?
Construct the finite field $\mathbb{F}_{27}$ as an extension of $\mathbb{F}_3 = \mathbb{Z}/3\mathbb{Z}$, by finding an irreducible polynomial of degree 3 with coefficients in $\mathbb{F}_3$. What are the three roots of your irreducible polynomial?
Let $\mathbb{Z}[x]$ be the ring of polynomials in $x$ with coefficients in $\mathbb{Z}$. Give an example of a maximal ideal $I \subset \mathbb{Z}[x]$. Give an example of an ideal $I$ which is prime but not maximal. Are there any ideals $I$ such that the quotient $\mathbb{Z}[x]$ is a field not of the form $\mathbb{Z}/p\mathbb{Z}$ for a prime $p$?