[1] Reduce the matrix $A = \begin{bmatrix} 30 & 30 & 30 \\ 30 & 10 & 30 \\ 30 & 30 & 6 \end{bmatrix}$ to diagonal form by integer row and column operations.
Define a principal ideal domain. State the ascending chain condition. Show that a principal ideal domain satisfies the ascending chain condition.
[3] Let \( F \subset K \subset L \) be fields. Prove that \([L : F] = [L : K][K : F]\).
Let $F \subset K \subset L$ be fields. Prove that if $L$ is algebraic over $K$ and $K$ is algebraic over $F$, then $L$ is algebraic over $F$. 

[4]
[5] Prove the Hilbert Basis Theorem: If a ring $R$ is noetherian, then so is the polynomial ring $R[x]$. 