[1] Define a ring homomorphism. Define an ideal. Define an integral domain. Prove that the kernel of a ring homomorphism is an ideal. When is the quotient of a ring homomorphism an integral domain? When is the quotient of a ring homomorphism a field?
[2] A message is represented as an integer $\alpha \mod 143$. You receive the encrypted message $\alpha^{11} \equiv 2 \mod 143$. What is $\alpha$?
Let $A$ be an $n \times n$ matrix with entries in $\mathbb{R}$, satisfying the polynomial relation
\[(x + 1)^2 (x + 2) = 0\]

Note that
\[(x + 1)^2 - x (x + 2) = 1\]

Find a formula for $e^{At}$ as a polynomial expression in $A$. 
[4] Construct the finite field $\mathbb{F}_8$ as an extension of $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$, by finding an irreducible polynomial of degree 3 with coefficients in $\mathbb{F}_2$. What are the three roots of your irreducible polynomial? Find a generator of the multiplicative group $\mathbb{F}_8^*$ of nonzero elements of $\mathbb{F}_8$. 
[5] Let $I = (2 + i)$ be the principal ideal generated by the element $2 + i$ in the Gaussian integers $\mathbb{Z}[i]$. Describe the quotient ring $\mathbb{Z}[i]/I$. 
