First Exam

Modern Algebra II, Dave Bayer, October 8, 2009

Name: ____________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[**1**] Define a ring homomorphism. Define an ideal. Define an integral domain. Prove that the kernel of a ring homomorphism is an ideal. When is the quotient of a ring homomorphism an integral domain? When is the quotient of a ring homomorphism a field?

[2] A message is represented as an integer $a \mod 143$. You receive the encrypted message $a^{11} \equiv 2 \mod 143$. What is a?

[3] Let A be an $n \times n$ matrix with entries in $\mathbb{R}$, satisfying the polynomial relation

$$(x+1)^2\,(x+2) \;=\; 0$$

Note that

$$(x+1)^2 \;-\; x\,(x+2) \;=\; 1$$

Find a formula for e^{At} as a polynomial expression in A.

[**4**] Construct the finite field $\mathbb{F}_8$ as an extension of $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$, by finding an irreducible polynomial of degree 3 with coefficients in $\mathbb{F}_2$. What are the three roots of your irreducible polynomial? Find a generator of the multiplicative group $\mathbb{F}_8^*$ of nonzero elements of $\mathbb{F}_8$.

[5] Let $I = (2+i)$ be the principal ideal generated by the element $2+i$ in the Gaussian integers $\mathbb{Z}[i]$. Describe the quotient ring $\mathbb{Z}[i]/I$.