Practice Final

Modern Algebra II, Dave Bayer, December 4, 2008

Name: ______________________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

[**1**] Prove the *Eisenstein Criterion*: If $f(x) \in \mathbb{Z}[x]$ and p is a prime, such that the leading coefficient of $f(x)$ is not divisible by p, every other coefficient of $f(x)$ is divisible by p, but the constant term is not divisible by p^2, then $f(x)$ is irreducible in $\mathbb{Q}[x]$.

[2] Show that $f(x) = x^5 - 16x + 2$ is irreducible in $\mathbb{Q}[x]$. Find a different degree 5 polynomial with exactly three real roots, that is also irreducible in $\mathbb{Q}[x]$.

[**3**] Prove the *Primitive Element Theorem*: Let K be a finite extension of a field F of characteristic zero. There is an element $\alpha \in K$ such that $K = F(\alpha)$.

[4] Which of the following cubic polynomials have A_3 for their Galois group? Which have S_3 for their Galois group?

$$x^3 - 2, \quad x^3 + 27x - 4, \quad x^3 + x + 1, \quad x^3 - 2x + 1, \quad x^3 + 3x + 14$$

[5] Let K be the splitting field over $\mathbb{Q}$ for the polynomial $f(x) = (x^2 - 2)(x^2 - 3)$. What is a primitive element for K over $\mathbb{Q}$? What is the Galois group of $f(x)$?