Exam 1

Modern Algebra II, Dave Bayer, October 2, 2008

Name: ______________________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[**1**] Define a ring homomorphism. Define an ideal. Prove that the kernel of a ring homomorphism is an ideal.

[2] Let A be an $n \times n$ matrix with entries in $\mathbb{R}$, satisfying the polynomial relation

$$(x-2)^3 \;=\; 0$$

Find a formula for e^{At} as a polynomial expression in A. Give an example of a matrix A for which this is the minimal polynomial relation, and check your formula using this matrix.

[3] Construct the finite field $\mathbb{F}_8$ as an extension of $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$, by finding an irreducible polynomial of degree 3 with coefficients in $\mathbb{F}_2$. What are the three roots of your irreducible polynomial?

[4] A message is represented as an integer $a \mod 35$. You receive the encrypted message $a^5 \equiv 3 \mod 35$. What is a?

[5] Give an example of a finite ring R which is not a field, such that $1 \neq 0$ but $1 + 1 + 1 = 0$.