

Practice Second Exam 1

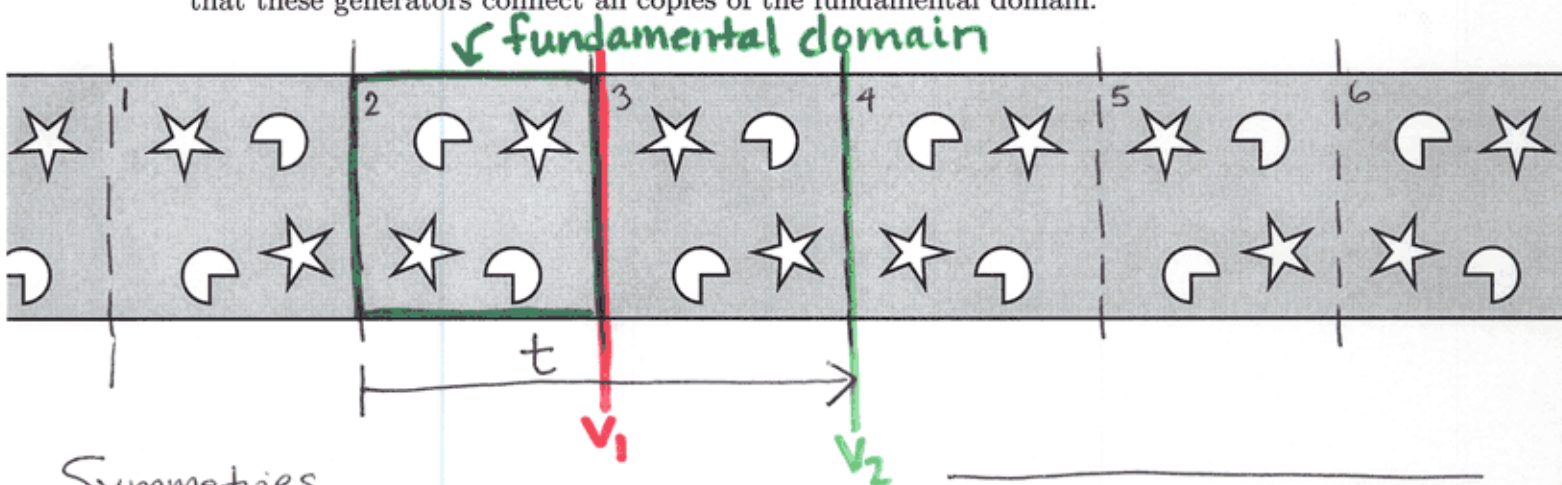
MATH V1010: Groups and Symmetry, Fall, 2003 (4, 6, 1, 7)

Name: _____ School: _____

[1]	[2]	[3]	[4]	TOTAL

You may use scratch paper, but only these pages will be graded. Do not hand in your scratch paper.

For each strip pattern, find a fundamental domain. Find the symmetries of the pattern. Draw another strip pattern with the same symmetries, using just the letter R. Choose a set of generators for the group of symmetries. Draw enough of a Cayley diagram to show that these generators connect all copies of the fundamental domain.



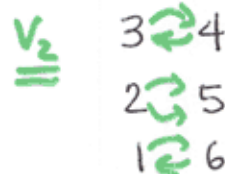
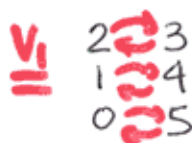
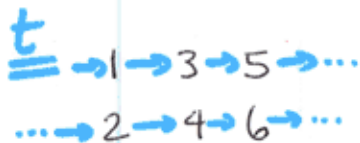
Symmetries

translations, vertical reflections

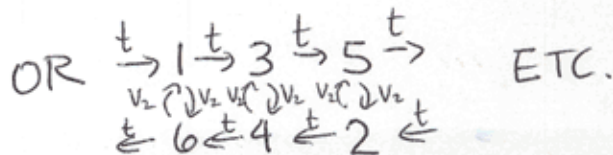
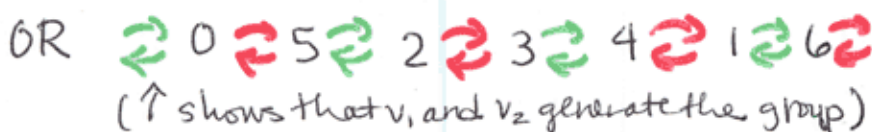
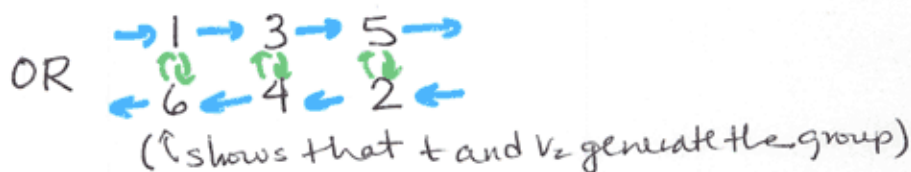
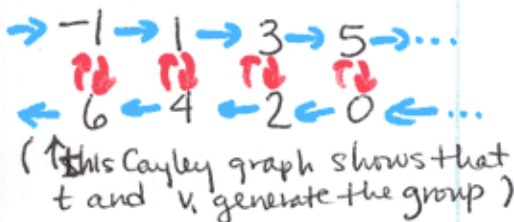
R R R R R R

Generators

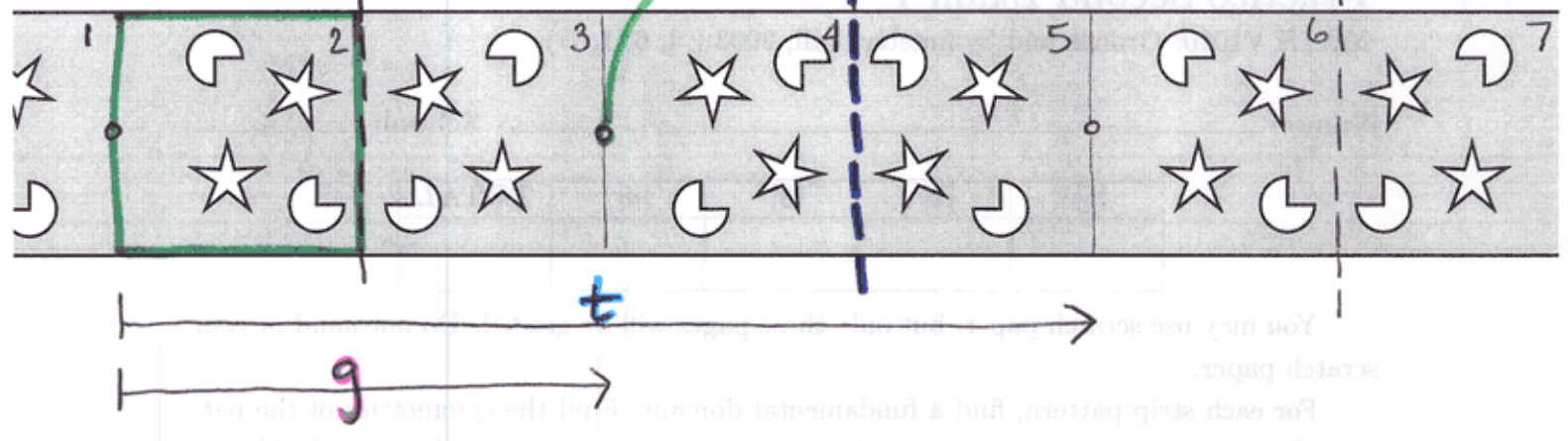
t, v_1, v_2



Cayley Graph



fundamental domain



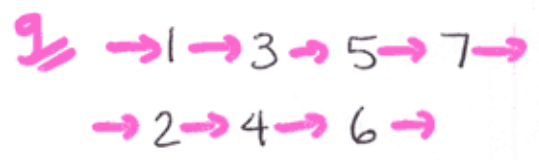
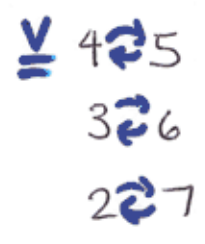
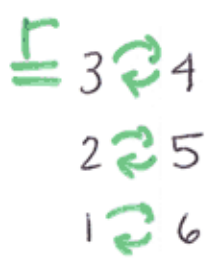
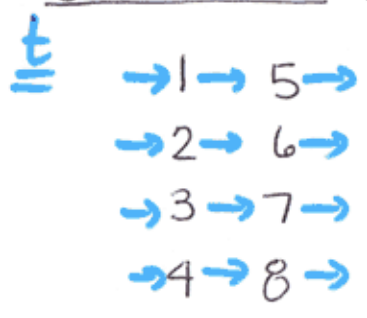
Symmetries

Translations, vertical reflections, rotations, glide reflections.

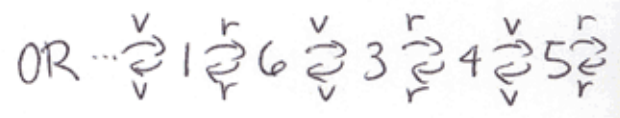
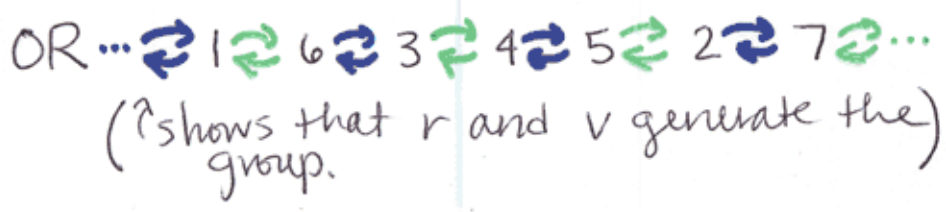
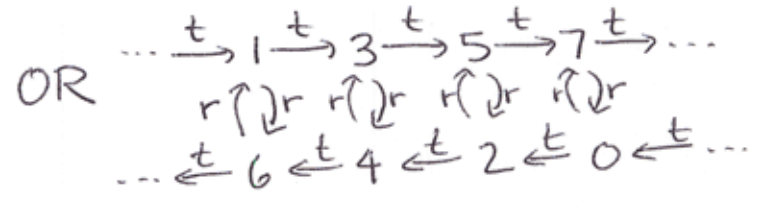
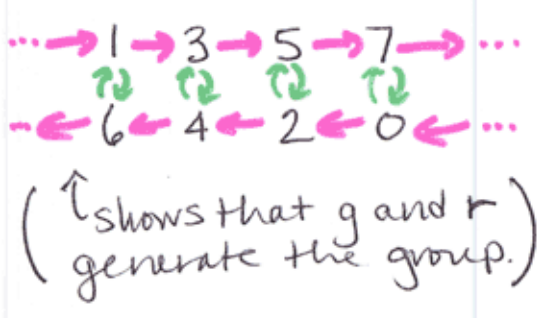
R R R R R R R R

Generators t, v, r, g

(Note: All rotations are equivalent. All vertical reflections are equivalent.)

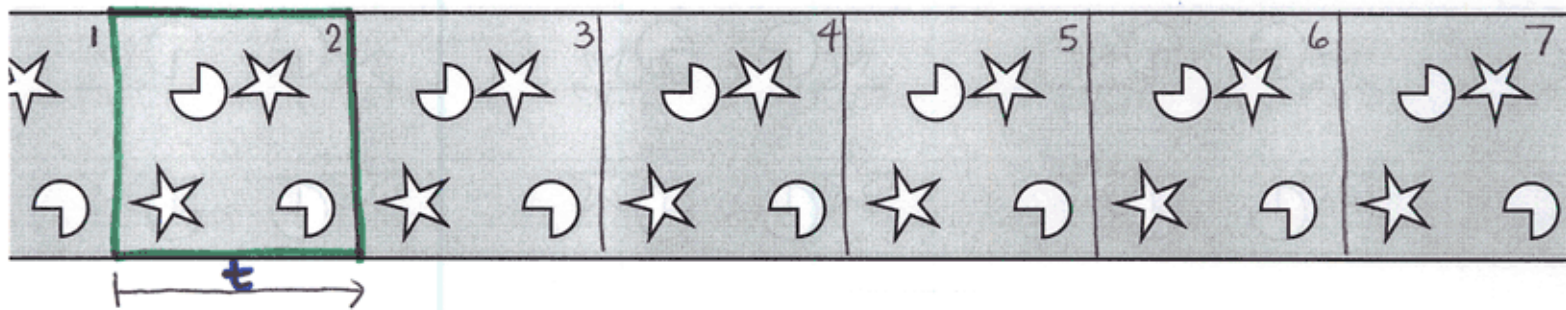


Cayley Graph

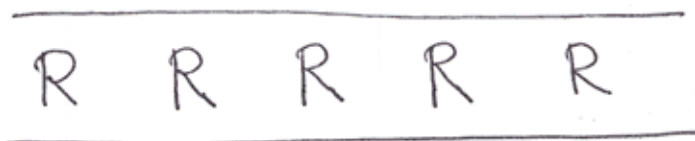


1.3

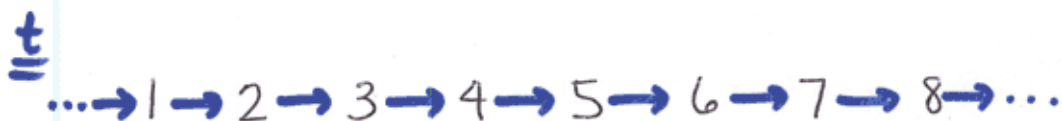
fundamental domain



Symmetries
Translations



Generators
t



Cayley Diagram



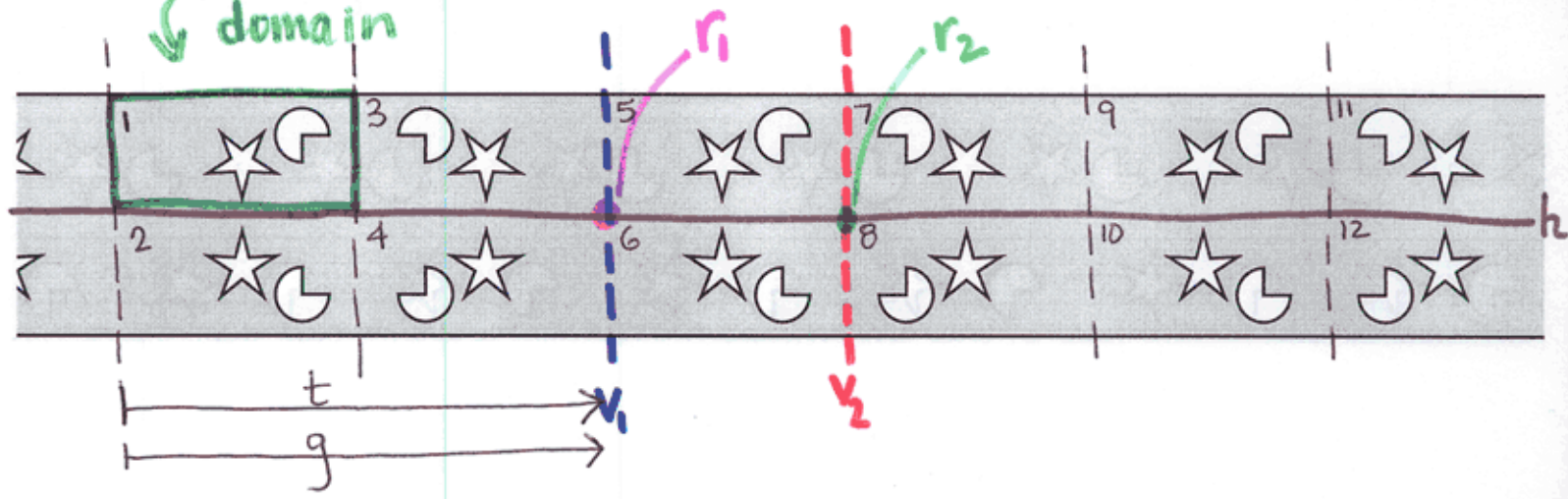
(shows that t generates the group.)

OR



1.4

fundamental domain



Symmetries

Translations, horizontal reflections, vertical reflections, rotations, glide reflections.

R	R	R	R	R	R	R	R	R	R
B	B	B	B	B	B	B	B	B	B

Generators

$t, r_1, r_2, v_1, v_2, h, g$

t

→ 1 → 5 → 9 →
 → 3 → 7 → 11 →
 → 2 → 6 → 10 →
 → 4 → 8 → 12 →

v₂

5 ↔ 7	4 ↔ 10
6 ↔ 8	1 ↔ 11
3 ↔ 9	2 ↔ 12

r₁

3 ↔ 6
4 ↔ 5
1 ↔ 8
2 ↔ 7

h

1 ↔ 2
3 ↔ 4
5 ↔ 6

r₂

5 ↔ 8	4 ↔ 9
6 ↔ 7	1 ↔ 12
3 ↔ 10	2 ↔ 11

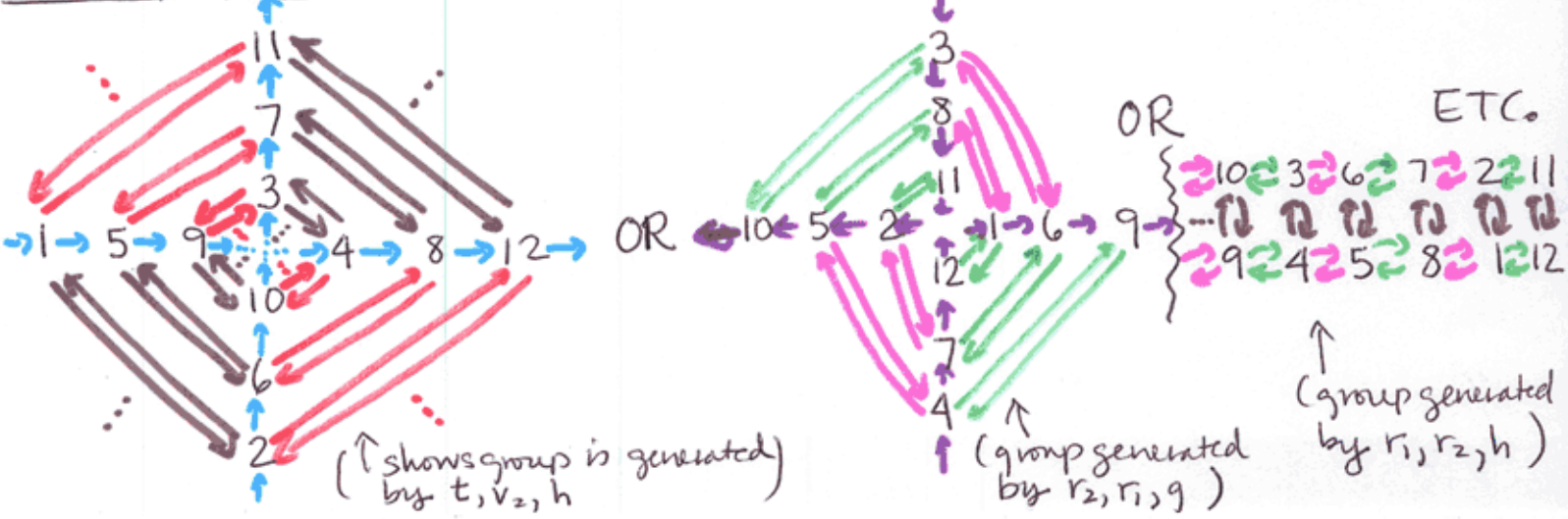
v₁

3 ↔ 5
4 ↔ 6
1 ↔ 7
2 ↔ 8

g

→ 1 → 6 → 9 →
 → 3 → 8 → 11 →
 → 2 → 5 → 10 →
 → 4 → 7 → 12 →

Cayley Graph



Practice Second Exam 2

MATH V1010: Groups and Symmetry, Fall, 2003 (1, 4, 5, 2)

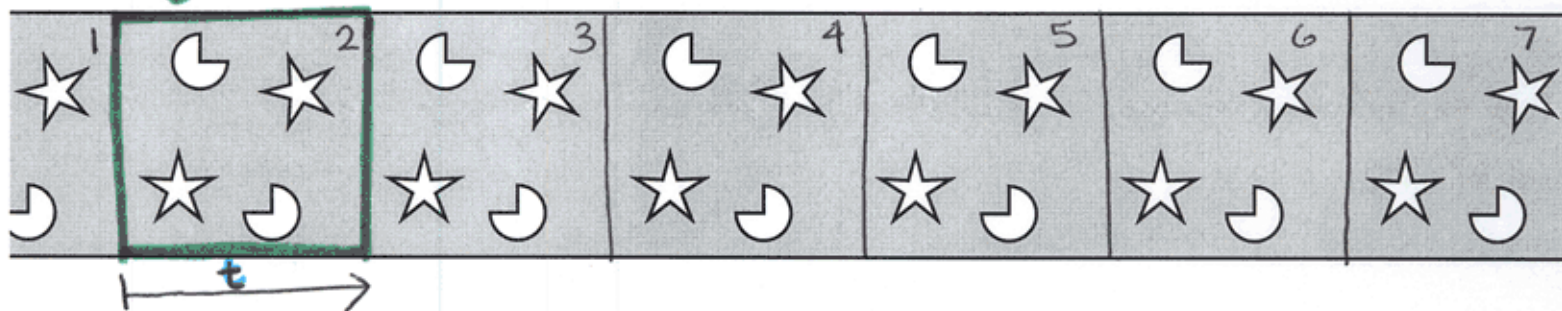
Name: _____ School: _____

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fundamental domain



Symmetries

Translations.

R R R R R

t ... → 1 → 2 → 3 → 4 → 5 → 6 → 7 → ...

Generators

t

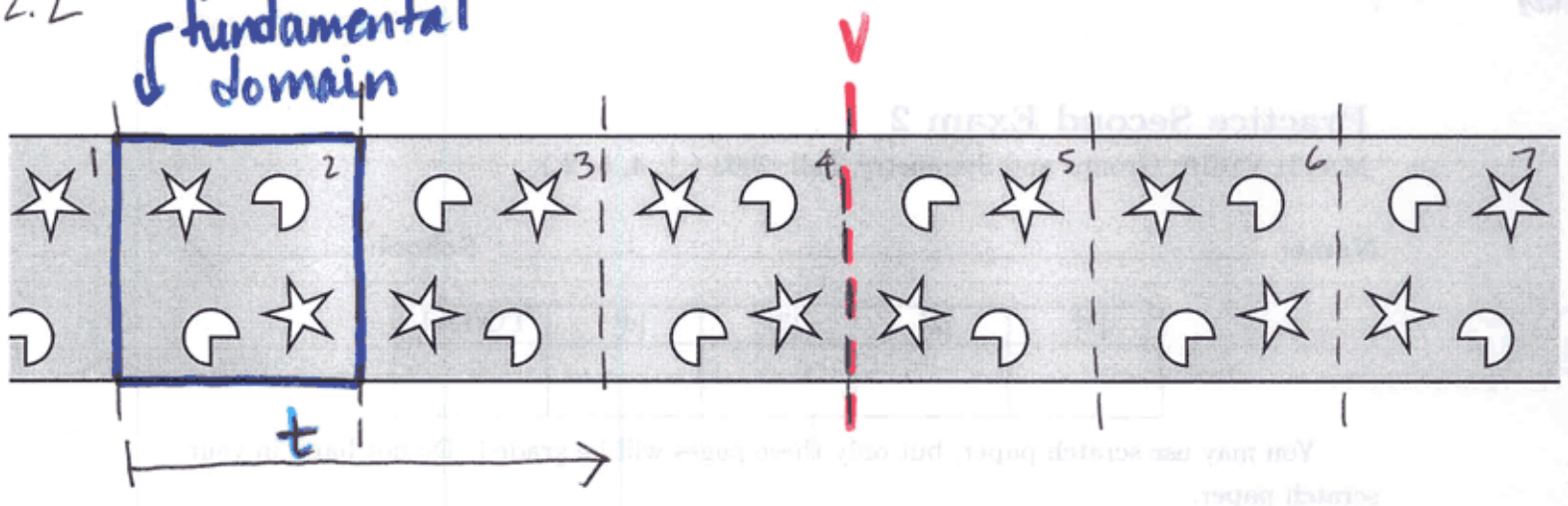
Cayley Graph

... → 1 → 2 → 3 → 4 → 5 → 6 → 7 → ...

(group is generated by t)

2.2

fundamental domain

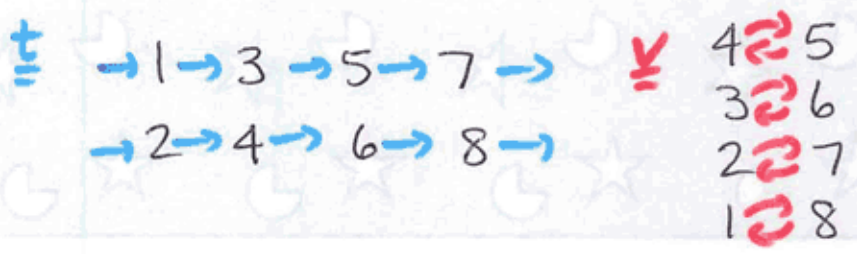


Symmetries

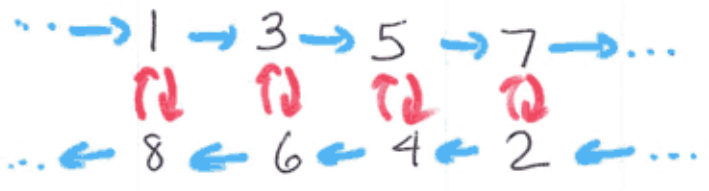
Translations, Vertical reflections

R R R R R

Generators t, v



Cayley graph



(group is generated by t, v)