

Practice First Exam AA

MATH V1010: Groups and Symmetry, September 25, 2003

Name: Solutions School: _____

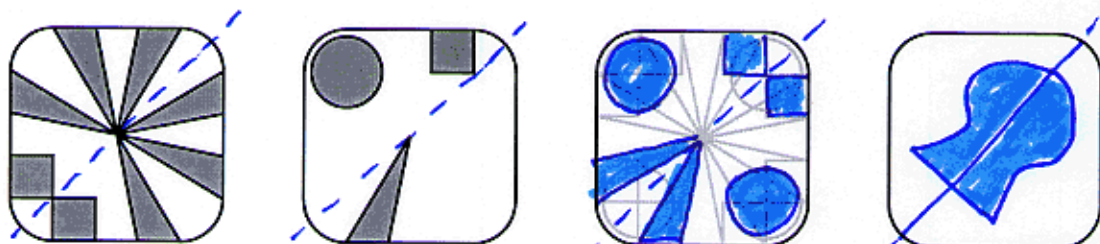
[1]	[2]	[3]	[4]	TOTAL

} please leave these blank

You may use scratch paper, but only this sheet will be graded; please present all answers on this sheet.

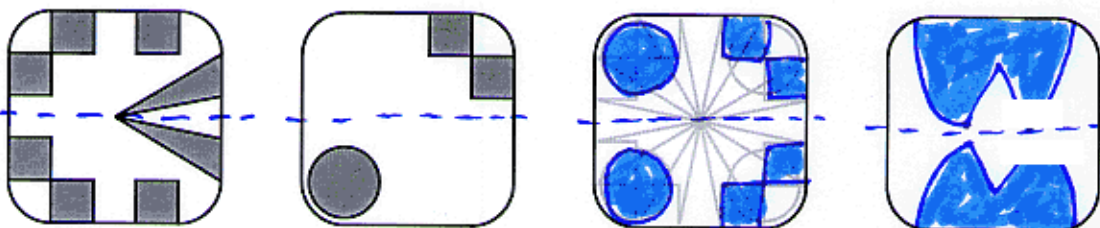
For problems [1] and [2], which symmetries of the square preserve the pattern on the left? Modify the second pattern so that it has the same subgroup of symmetries. In the blank square on the right, design your own pattern with the same subgroup of symmetries.

[1]



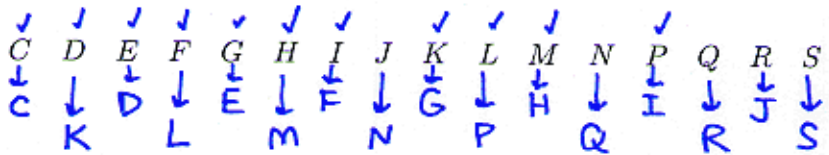
{ 1, ~~σ~~ }

[2]



{ 1, σ }

[3] What is the cycle structure of a perfect shuffle of the deck of 16 cards shown? (Use the names of the cards shown. Do not use numbers 1, 2, ...) How many perfect shuffles does it take for the deck to return to its starting position?



periods 1, 4, 4, 2, 4, 1 4 shuffles

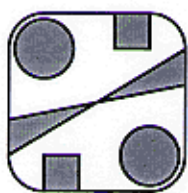
AA

[4] Under each pattern, write the symmetries of the square that transform the leftmost pattern to each pattern. Use these actions as row and column labels for the multiplication table, and fill in the multiplication table. Show the quotient group structure.



A

1 ↻



B

↻ ↻



C

↻ ↻



D

↻ ↻

A {

1	↻	↻ ↻	↻ ↻	↻ ↻	↻ ↻
↻	1	↻ ↻	↻ ↻	↻ ↻	↻ ↻

B {

↻	↻	1	↻	↻ ↻	↻ ↻
↻	↻	↻	1	↻ ↻	↻ ↻

C {

↻	↻	↻ ↻	↻ ↻	1	↻
↻	↻	↻ ↻	↻ ↻	↻	1

D {

↻	↻	↻ ↻	↻ ↻	↻ ↻	↻ ↻
↻	↻	↻ ↻	↻ ↻	↻ ↻	↻ ↻

	A	B	C	D
A	A	B	C	D
B	B	A	D	C
C	C	D	A	B
D	D	C	B	A

Practice First Exam AB

MATH V1010: Groups and Symmetry, September 25, 2003

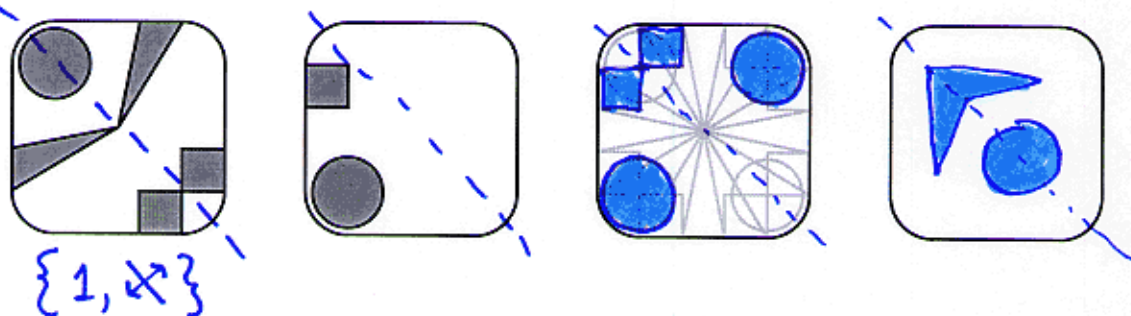
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[1]	[2]	[3]	[4]	TOTAL

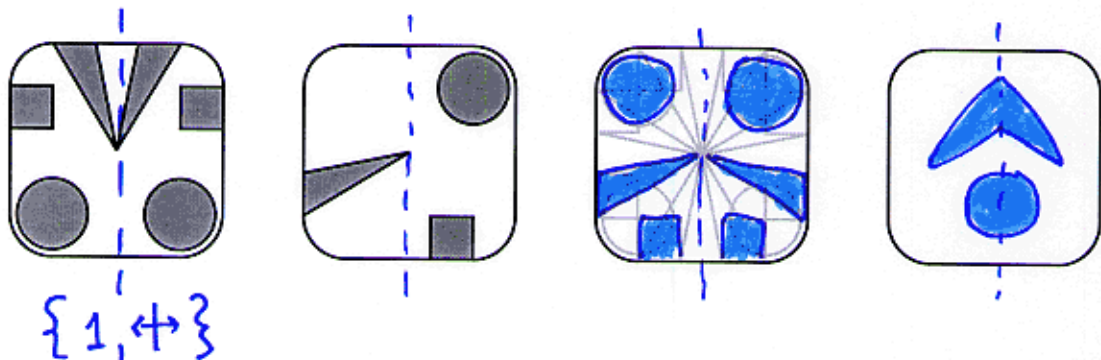
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[1]



[2]



[3] What is the cycle structure of a perfect shuffle of the deck of 16 cards shown? (Use the names of the cards shown. Do not use numbers 1, 2, ...) How many perfect shuffles does it take for the deck to return to its starting position?

$\begin{matrix} \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow \\ E & F & G & H & I & J & K & L & M & N & P & Q & R & S & T & U \\ \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow \\ E & M & I & G & H & N & R & K & J & P & L & Q & S & T & U & U \end{matrix}$

$(E)(FMIG)(HNRK)(JP)(LQST)(U)$
 $\quad 1 \quad 4 \quad 4 \quad 2 \quad 4 \quad 1$

4 shuffles

AB

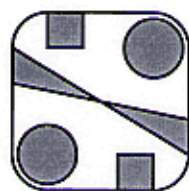
[4] Under each pattern, write the symmetries of the square that transform the leftmost pattern to each pattern. Use these actions as row and column labels for the multiplication table, and fill in the multiplication table. Show the quotient group structure.



A



B



C



D

1 ↻

↻ ↺

↗ ↘

↔ ↕

A {
1
↻

1	1	↻	↻	↺	↺	↗	↘	↔	↕
↻	↻	1	↺	↺	↺	↘	↘	↕	↔

B {
↻
↺

↻	↻	↺	↻	1	1	↔	↕	↘	↘
↺	↺	↻	1	↻	↻	↕	↔	↘	↘

C {
↗
↘

↗	↗	↘	↕	↔	1	↻	↺	↺	↺
↘	↘	↗	↔	↕	↻	1	↻	↺	↺

D {
↔
↕

↔	↔	↕	↘	↘	↻	↺	1	↻	↺
↕	↕	↔	↘	↘	↺	↻	↻	1	↺

	A	B	C	D
A	A	B	C	D
B	B	A	D	C
C	C	D	A	B
D	D	C	B	A

Practice First Exam AC

MATH V1010: Groups and Symmetry, September 25, 2003

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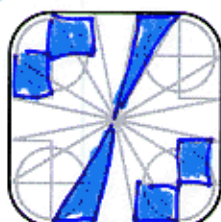
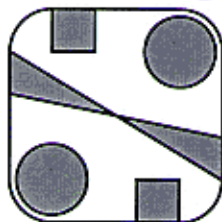
[1]	[2]	[3]	[4]	TOTAL

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[1]

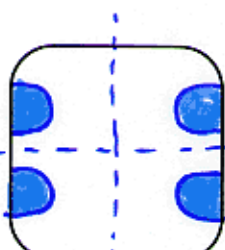
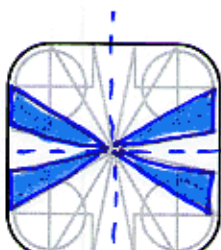
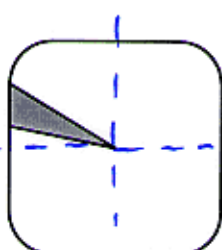
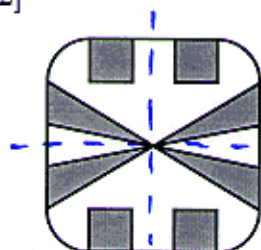
↻ clockwise from square to circle



{1, ↻}

(notion of clockwise ↻ so no flips)

[2]



{1, ↻, ↕, ⇄}

[3] What is the cycle structure of a perfect shuffle of the deck of 16 cards shown? (Use the names of the cards shown. Do not use numbers 1, 2, ...) How many perfect shuffles does it take for the deck to return to its starting position?

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓
 B C D E F G H I J K L M N P Q R
 B J C K D L E M F N G P H Q I R

(B)(C J F D)(E K N H)(G L)(I M P Q)(R)
 1 4 4 2 4 1

4 shuffles