Project guidelines

Calculus II, section 3

1. Timeline and requirements

You may choose to replace the final exam with a final project, either individual or in groups of up to 3; group projects will need to be correspondingly in greater depth and scope. The goal of each project is to 1) investigate some mathematical problem using the concepts we've studied in this class, 2) write an expository paper on the topic, i.e. explain it in detail to an audience unfamiliar with it, and 3) give a short presentation to the class on your topic at the end of the semester. Depending on the project, your exposition may also involve computer simulations, in which case the expository paper might (if appropriate) be correspondingly shorter but should explain these simulations.

This is a fairly open-ended project; you may use any resources you like, and the converse of this is that it is your job to find (and properly cite) references to understand your desired topic. That said, I am happy to help you find resources if you are having difficulty, especially on more obscure topics.

Timeline

The deadlines are as follows.

March 30 Deadline to let me know via email if you would like to do a project, with your first and second choice of topic (see below). (If you are working in a group, only one group member needs to email me, including the names of their collaborators.)

April 11 Deadline to submit an outline (at most one page) of what you plan to do and what sources you intend to use, including examples and exercises. This should be thought of as a rolling deadline: the sooner you submit your outline, the sooner you will get feedback on it, which you may then need to incorporate:

April 13 Deadline to resubmit your outline incorporating feedback from the previous draft, if necessary.

April 20 Deadline to schedule a meeting to discuss (and, if desired, practice) your presentations, which are tentatively scheduled for April 25.

May 2 Deadline to turn in an *optional* first draft to get feedback on it before submitting the final draft.

May 11 Tentative due date for the final project; this may be adjusted by a day or two once the final exam schedule is published.

Guidelines

The primary goal of this project is to understand the mathematics of your topic; nearly as important however is clearly communicating that understanding. Imagine that you are trying to explain the concepts you have studied to someone who took a calculus 2 course some years in the past and so vaguely recalls the material but may need you to review the more complicated topics.

Like any paper, in addition to the main body of the exposition your paper should include an introduction, explaining the main ideas, motivation, and background of your paper, as well as a list of sources. (The specific formatting of your sources does not matter so long as it is clear.) For group projects, in addition to the main paper, each group member must separately submit a short description (a few sentences at most) of their main contributions to the project.

All papers should be typed.[1] I encourage you to use LaTeX[2], and am happy to hold workshops on it if there is interest; however it is not required, and you may use whatever software you prefer.

There is no hard guideline for the length of your papers: they should be the length they need to be in order to concisely and clearly explain your topic in detail to the appropriate audience, but in practice probably individual papers should be at least 2-3 pages, and e.g. 3-person papers at least 5-6.

Grading

The steps outlined above (outline and meeting) are required, but graded only for completion, and similarly a rough draft (if turned in) is only given comments, not graded. Completion of the mandatory steps is worth 5% of the project grade. The remaining grade is as follows:

- mathematical correctness: 50%
- clarity and quality of exposition: 30%
- depth, style, creativity: 15%

2. Topic suggestions

Any of these should be taken as a collection of related possible ideas around which to base your project; you do not necessarily need to cover everything mentioned, and might cover aspects not mentioned.

Higher-order Newton's method. We saw in class how Newton's method, which works by repeatedly using first-order approximation to a function, can be used to solve differential equations; it can also be used to find zeros of general functions. For various reasons, it does not always work; even if it does, as we saw in class it often converges quite slowly. It is possible to improve this method using higher-order approximations, i.e. Taylor polynomials. Explain this method, and give some examples to see how it compares with the usual Newton's

[1] If this is a particular hardship for you, we can discuss alternatives.

[2] LaTeX is a software system for creating documents, especially those involving large numbers of mathematical or scientific symbols, and is probably what virtually all mathematical documents you have encountered at least in college were written in, including this one; there are many editors available, including online ones such as `overleaf.com`.

method, and why one or the other might be preferable. Depending on your interests, you could also look into conditions of convergence for both higher-order and usual Newton's method, or try and estimate the error under appropriate conditions.

Differential equations via Taylor series. Another method to solving differential equations, at least for *analytic* functions, is to write our function as a Taylor series and reinterpret the differential equation as a condition on the coefficients, which may then be easier to solve. In some cases, this may let us fully describe solutions to differential equations as Taylor series which we don't know how to describe any other way. Explain how this works, and illustrate via some examples. You might also talk about situations in which this can't work, or isn't the best approach.

Generating functions. Often, e.g. in combinatorics or number theory, we have a sequence $\{a_n\}$ with some relations, e.g. recurrences or more complicated equations relating different terms, that we want to study; a classic example is the Fibonacci sequence $a_0 = a_1 = 1$, $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$, so that $a_2 = 2$, $a_3 = 3$, $a_4 = 5$, $a_5 = 8$, etc. One approach to studying such things is to define a power series with coefficients $\{a_n\}$, or a few different but similar schemes; we can then study this power series as a function, and use its properties to recover information about our sequence. Explain how this works and give some examples. This is a fairly large field; you could also investigate particular types of methods in more detail.

Fourier series. Taylor series are not the only way to write suitable functions as (infinite) sums of simpler functions; another is Fourier series, which instead of power series represents functions as sums of trigonometric functions. Explain how this works, and discuss some of its many applications.

Divergent series. We can only describe convergent series by a number, namely the limit of their partial sums. However, even if a series diverges, in some cases it is still possible to associate to it a number, which is not equal to the sum (since it does not converge) but is in a certain sense the "next best thing." This is the origin of some apparently nonsensical "identities," such as $1 - 1 + 1 - 1 + \cdots = \frac{1}{2}$ or $1 + 2 + 3 + 4 + \cdots = -\frac{1}{12}$. Investigate some such methods and explain the underlying ideas, with appropriate examples.

Choose your own. Propose your own topic! As a final project, it should be based on the material from this class, and in particular it should involve the material from the final unit, sequences and series, which otherwise has not been assessed. Otherwise you are free to choose any topic that interests you, using the above as a guide.