Homework 7

Calculus II, section 3

Hard due date: 6:10 PM Wednesday March 23, 2022
+2 extra credit points if turned in by 6:10 PM Tuesday March 22
+5 extra credit points if turned in by 6:10 PM Monday March 21

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

Any error in either the lecture notes or the homework is worth up to 5 points of extra credit to the first person to spot it, depending on the severity of the error; email me (cbz2106@columbia.edu) if you find one. (You do *not* lose points for incorrectly pointing out an error, so please do not hesitate!)

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

Problem 1. Consider the first-order equation $y' = 3x^2y$.

(a) Find a one-parameter family of solutions for $y(x)$. (15 pts)

(b) Find a solution satisfying equation and the initial condition $y(0) = 1$. (5 pts)

(c) Show that your solution to part (b) is the only solution to the equation with $y(0) = 1$. (15 pts)

Problem 2. Consider a mass m on a spring with spring constant k, like we looked at in class, but now the mass is moving with some amount of air resistance, which pushes back in the opposite direction to the motion with force proportional to the velocity, according to some positive coefficient c. In other words, in addition to the force $-kx$ depending on the position x as a function of time, we also have a force $-cx'$ depending on the velocity x'. Combining with Newton's second law, we have the equation

$$F = ma = mx'' = -cx' - kx.$$

This is a second-order differential equation.

(a) Suppose that c is very small relative to m and k, so there is very little air resistance. Assuming initial position x_0 and initial velocity v_0, find the position x as a function of t. (15 pts)

(b) Now suppose that c is very large relative to m and k, i.e. a lot of air resistance, and again find the position x assuming initial position x_0 and velocity v_0. (15 pts)

(c) Recalling the "types" of second-order equations of this form from class, what is the point (i.e. the value of c in terms of k and m) at which the behavior of the equation switches from that described in (a) to that in (b)? (5 pts)

Problem 3. Find a solution to the second-order equation $x'' - 2x' + x = 2\sin t$ with $x(0) = x'(0) = 0$. (30 pts)

Survey (optional). Complete the following survey by rating each problem you attempted on a scale of 1 to 10 according to how interesting you found it (1 = "mind-numbing," 10 = "mind-blowing"), and how difficult you found it (1 = "trivial," 10 = "brutal"). Also estimate the amount of time you spent on each problem to the nearest half hour.

	Interest	Difficulty	Time Spent
Problem 1			
Problem 2			
Problem 3			

Please feel free to record any additional comments you have on the problem sets and the lectures, in particular, ways in which they might be improved.