Homework 5

Calculus II, section 3

Hard due date: 6:10 PM Wednesday March 2, 2022

+2 extra credit points if turned in by 6:10 PM Tuesday March 1

+5 extra credit points if turned in by 6:10 PM Monday February 28

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

Any error in either the lecture notes or the homework is worth up to 5 points of extra credit to the first person to spot it, depending on the severity of the error; email me (cbz2106@columbia.edu) if you find one. (You do *not* lose points for incorrectly pointing out an error, so please do not hesitate!)

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

Problem 1. Consider the parametric curve given by $x = \cos^2 t$, $y = \sin^2 t$ for t between 0 and π.

(a) Show that this is the segment of the line $y = 1 - x$ between $x = 0$ and $x = 1$. (5 pts)

(b) Compute the arc length of the curve. (10 pts)

(c) Why is this arc length not equal to $\sqrt{2}$? (5 pts)

Problem 2. Consider the curve $y = e^x$.

(a) Find the length of the curve between $x = 0$ and $x = N$ (assuming $N > 0$). (20 pts)

(b) Compare this arc length to the length of a straight line between the endpoints $(0, 1)$ and (N, e^N). Describe what happens as $N \to \infty$. (15 pts)

Problem 3. Above the Arctic Circle (for the purposes of this problem, the circle of latitude at 66°, or $\frac{11\pi}{30}$ radians, above the equator) and below the Antarctic Circle (at latitude 66° below the equator) there is always at least one 24-hour period where the sun is always visible. On what proportion of the Earth's surface is this true? In other words, what proportion of the surface of the Earth is in either the Arctic or Antarctic regions? (25 pts)

Problem 4. Consider the spiral given in polar coordinates by $r = e^{-\theta}$, for θ ranging from 0 to infinity. Does the improper integral for the total length of this spiral over this infinite interval converge? If so, what is it? (20 pts)

Survey (optional). Complete the following survey by rating each problem you attempted on a scale of 1 to 10 according to how interesting you found it (1 = "mind-numbing," 10 = "mind-blowing"), and how difficult you found it (1 = "trivial," 10 = "brutal"). Also estimate the amount of time you spent on each problem to the nearest half hour.

	Interest	Difficulty	Time Spent
Problem 1			
Problem 2			
Problem 3			
Problem 4			

Please feel free to record any additional comments you have on the problem sets and the lectures, in particular, ways in which they might be improved.