Homework 2

Calculus II, section 3

Hard due date: 6:10 PM Wednesday February 2, 2022
+2 extra credit points if turned in by 6:10 PM Tuesday February 1
+5 extra credit points if turned in by 6:10 PM Monday January 31

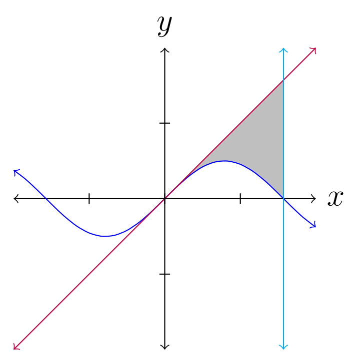
As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

Any error in either the lecture notes or the homework is worth up to 5 points of extra credit to the first person to spot it, depending on the severity of the error; email me (cbz2106@columbia.edu) if you find one. (You do *not* lose points for incorrectly pointing out an error, so please do not hesitate!)

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

Problem 1.

(a) Find the area of the region bounded by the curves $y = \sin(x)\cos(x)$ and $y = x$ and the line $x = \frac{\pi}{2}$. (10 pts)



(b) Consider the solid formed by rotating about the y-axis the region bounded by the x-axis, the y-axis, and the curve $y = \sqrt{1 - x^2}$. Using the method of cylindrical shells, find the volume of this solid. What is this solid, geometrically? Does your answer make sense? (10 pts)

Problem 2. In class, we found an antiderivative of $\log x$ by knowing the antiderivative of e^x. Generalize that method to find an antiderivative for $\sin^{-1}(x)$, the inverse function of $\sin(x)$, wherever it is defined (20 pts).

Problem 3. Consider a (solid) sphere of radius R, and consider its volume as an integral of the areas of layers, each of which is a disk of some radius r. Let x be the coordinate indexing the layers, so that the layer at $x = 0$ is a disk of radius R, and as x approaches either R or $-R$ the corresponding layer shrinks to a point.

(a) Find a formula for the radius of the layer at x, and use this to give a formula for the area of the layer at x. (5 pts)

(b) Use your answer to part (a) to set up and compute an integral for the volume of the sphere of radius R. (10 pts)

(c) The surface area of a sphere of radius R can be thought of as the infinitesimal change in the volume as the radius increases: if we take our sphere and subtract a slightly smaller sphere, we are left with a spherical shell. Taking the thickness of this shell to 0, we get that the surface area is the derivative of the volume with respect to R. Use this together with your answer to part (b) to compute the surface area of a sphere of radius R. (5 pts)

(d) Verify your answer to part (c) by using it to again compute the volume of a solid sphere of radius R: we can think of our sphere as gluing together spherical shells of every radius less than R, so we can compute the volume of the whole sphere as the integral of these areas as the radius varies from 0 to R. Compute the volume in this way and verify that this matches your answer to part (b). (5 pts)

Problem 4. Compute the definite integral $\int_e^{e^2} \frac{(\log(\log x))^2}{x}\,dx$. (15 pts)

Problem 5. Compute the indefinite integral $\int \sin(\log x)\,dx$ where $x > 0$. (15 pts)

Survey (optional). Complete the following survey by rating each problem you attempted on a scale of 1 to 10 according to how interesting you found it (1 = "mind-numbing," 10 = "mind-blowing"), and how difficult you found it (1 = "trivial," 10 = "brutal"). Also estimate the amount of time you spent on each problem to the nearest half hour.

	Interest	Difficulty	Time Spent
Problem 1			
Problem 2			
Problem 3			
Problem 4			
Problem 5			

Please rate each of the following lectures that you attended, according to the quality of the material (1="useless", 10="fascinating"), the quality of the presentation (1="epic fail", 10="perfection"), the pace (1="way too slow", 10="way too fast", 5="just right") and the novelty of the material to you (1="old hat", 10="all new").

Date	Lecture Topic	Material	Presentation	Pace	Novelty
1/24	Integrals and volume				
1/26	Integration by parts				

Please feel free to record any additional comments you have on the problem sets and the lectures, in particular, ways in which they might be improved.