Title: Topological Insulators and the Bulk-Edge Correspondence using Fredholm theory

Project Leader: Jacob Shapiro

Abstract: In mathematical physics, the bulk-edge correspondence for topological insulators refers to the fact that certain topological invariants (e.g., Fredholm indices of Fredholm operators on Hilbert space) which correspond to physical quantities (e.g., Hall conductivity) computed from a Hamiltonian working on infinite space (bulk) or from the associated restriction of that Hamiltonian to the half-infinite space (edge) are equal. This is a highly non-trivial result given that these correspond to different physical phenomena. There are many settings and contexts in which one can prove this correspondence, which also relates back to other correspondences in math and physics (Atiyah-Singer, AdS-CFT just to name two).

Our goal will be to explore some of these examples from in the simplest non-trivial setting (both mathematically and physically) which means studying Hamiltonians of non-interacting electrons using standard tools of functional analysis and topology. After picking up the basic tools and language, we will cover some known proofs and ultimately hopefully prove the correspondence in a new setting.

We will proceed in the following steps:

0) General introduction to topological insulators from the perspective of quantum mechanics of non-interacting electrons.
1) Study the correspondence in the chiral 1D spectrally gapped discrete case, in which the proof reduces to simple facts about Fredholm theory.
2) Study the correspondence in IQHE (integer quantum Hall effect, 2D) spectrally gapped discrete case, first via functional analysis (Graf-Elbau 2002), then using K-theory (Kellendonk, Richter, Schulz-Baldes 2002).
3) We will consider the (yet unsolved) problem for the Fu-Kane-Mele time-reversal invariant 2D system.
4) We will make a perfunctory overview of the Graf-Porta 2012 solution to this problem in the restricted translation-invariant setting.
5) In this last part we will try to produce yet another proof of item 2), now however via homotopy properties of the Fredholm index, trying to imitate what we learnt in item 1). Should we succeed, this will give an immediate solution of the unsolved

item 3), via the analogous homotopy properties of the Skew-adjoint Fredholm index.

Since this project lies at the intersection between math and physics, ideal candidates should have a background in both subjects. Helpful background in math includes: functional analysis, algebraic topology and Fredholm theory. Helpful background in physics includes: quantum mechanics, solid state physics.

If you lack the background in one of these areas I could give you some guidance towards getting up to speed before the program officially begins. It would probably be easier to make up for lack of math than lack of physics, but strong math candidates who lack the physics background will be considered.