Title: Curves on surfaces over finite fields

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Abstract: An algebraic variety is a geometric object given as the solution set to a system of polynomial equations. In this project, we will study two-dimensional algebraic varieties (surfaces) defined by a single polynomial with coefficients in a finite field. Our goal is to understand the one-dimensional algebraic varieties (curves) that lie on such a surface. In particular, we aim to construct surfaces which contain "as many linearly independent curves as possible"; examples of such surfaces are sparse, and it would be quite interesting to have more. By some deep theorems and conjectures in algebraic geometry, this problem can be translated into a more concrete one involving numbers of solutions to the defining polynomial over finite fields. This more concrete problem is the one we will try to tackle.