

Rough paths theory and Gaussian processes

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Abstract

The theory of rough paths goes back to a series of articles written by T. Lyons in the 90s. Lyons' goal (which he eventually achieved) was to establish a theory rich enough to solve stochastic differential equations (SDEs) pathwise. Since then, rough paths theory has been further developed in several directions (for example by P. Friz, M. Gubinelli, M. Hairer,...) and is, by now, a well established theory which has applications in various fields of stochastic analysis and stochastic partial differential equations (SPDEs).

We start our talk with a brief introduction to the theory of rough paths. Then we explain how Gaussian processes can be viewed as rough paths in the sense of Friz–Victoir. We give some examples such as the fractional Brownian motion (a generalization of the Brownian motion which is *not* a martingale) and a spatial process arising from the stochastic heat equation (which was used by Hairer for solving a non-linear SPDE in [Hairer, CPAM 64, no.11 (2011), 1547-1585]). In the third part of the talk we present two applications. The first one is concerned with numerical simulations for solving SDEs driven by Gaussian signals. To be precise, we present a numerical scheme which is easy to implement in practice and give an upper bound for its rate of convergence. The second application involves the concentration of measure phenomenon on path spaces. We show how rough paths theory may be used to establish so-called transportation-cost inequalities for the law of solutions to SDEs driven by Gaussian processes.