MATH V1201 SECTIONS 002 & 003 HOMEWORK 5
DUE MARCH 11, 2015

ROBERT LIPSHITZ

1. Review

(I.1) The Earth moves in an ellipse with orbital eccentricity 0.0167 and minimum distance from the sun 147×10^6 kilometers. Write a polar equation for the Earth's orbit.[1]

2. Some Stewart problems

(II.1) Stewart 13.4.5.
(II.2) Stewart 13.4.30.
(II.3) Stewart 13.4.35.
(II.4) (Optional:) Stewart 13.3.59.

3. More practice

(III.1) Find a curve $\vec{r}(t)\colon \mathbb{R} \to \mathbb{R}^3$ so that $\vec{r}''(t)$ is non-zero but $\vec{T}'(t) = 0$.
(III.2) Find the normal plane and osculating plane to $\vec{r}(t) = \langle \cos(t^3), \sin(t^3)/\sqrt{2}, \sin(t^3)/\sqrt{2} \rangle$ at $t = (\pi/3)^{1/3}$.
(III.3) (Optional) Find the normal plane and osculating plane to $\vec{r}(t) = \langle 3\cosh(t), 4\sinh(t), 3t \rangle$ at $t = 1$. (Hint: look up some identities for cosh and sinh to make your computations easier.)
(III.4) Compute the curvature of $\vec{r}(t) = \langle 3\cosh(t), 4\sinh(t), 3t \rangle$.
(III.5) What curves have curvature identically equal to 0? Justify your answer with precise mathematics, not fuzzy geometric reasoning.

4. Mathematica

(IV.1) Using Mathematica's `PolarPlot` function, plot the Earth's orbit from Problem (I.1), and also, on the same plot, a circular orbit of radius 147×10^6 kilometers. (Surprised?)
(IV.2) Mathematica can sometimes only simplify expressions if you tell it something about the variables. For example, try the following:
  (a) `Sqrt[x^2]-x`
  (b) `Simplify[Sqrt[x^2]-x,x>0]`
  (c) `Norm[{Sin[t], Cos[t]}]`
  (d) `Simplify[Norm[{Sin[t], Cos[t]}], Element[t, Reals]]`

(In the last case, you're telling Mathematica that t is a real number, not a complex number.)
(IV.3) Use Mathematica to check your answers to questions (III.2) and (III.4).
(IV.4) (Optional:) Use Mathematica to plot the curves from part III, and check that your answers look reasonable.
(IV.5) (Optional:) Read the definition of the osculating circle (Stewart, p. 859). Then write Mathematica code which animates the osculating circle to $\vec{r}(t) = \langle t, t^2, t^3 \rangle$, as t runs from 0 to .5 (say).

E-mail address: `lipshitz@math.columbia.edu`

[1] Data for this problem comes from Earth. (2015, February 25). In Wikipedia, The Free Encyclopedia. Retrieved 22:30, February 28, 2015, from `http://en.wikipedia.org/w/index.php?title=Earth&oldid=648767870`.