MATH V1201 SECTIONS 002 & 003 HOMEWORK 10
DUE MAY 4, 2015

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1. Some Stewart problems

(I.1) 14.8.16.
(I.2) 14.8.19.
(I.3) 14.8.25.
(I.4) 14.8.26.
(I.5) 14.8.23.

2. More Lagrange multipliers

(II.1) Consider finding maxima and minima of the function $f(x, y) = x^2 + (y + 1)^2$ on the curve C given by $x^2 - \frac{1}{3}y^3 = 0$.

(a) Plot the curve C—this is Exercise (IV.1)
(b) Use your plot and thinking, not calculus, to answer: does $f(x, y)$ have a global maximum on C? If so, where / why? If not, why not?
(c) Use your plot and thinking, not calculus, to answer: does $f(x, y)$ have a global minimum on C? If so, where / why? If not, why not?
(d) Use Lagrange multipliers to try to find the global maximum and minimum of $f(x, y)$. Something should go wrong. What? Why is this possible?

3. A little more limit practice

(III.1) Does

$$\lim_{(x,y)\to(-1,1)} \frac{x^2 - y^2 + 2x + 2y}{x^2 + 2x + 1 + y^2 - 2y + 1}$$

exist? Prove your answer using the squeeze theorem, polar coordinates trick, or approaching along different paths, as appropriate.

(III.2) Consider the function

$$f(x, y) = \begin{cases} \frac{x^4}{x^2+\sin(y)^2} & \text{if } x^2 + \sin(y)^2 \neq 0 \\ 0 & \text{if } x^2 + \sin(y)^2 = 0. \end{cases}$$

At what points (x, y) is $f(x, y)$ continuous? Prove your answer. (There will be several different cases to consider.)

4. Mathematica

(IV.1) Plot the curve C from Problem (II.1).

(IV.2) Graph the functions from Problems (III.1) and (III.2), as sanity checks for your answers. Choose suitable regions to plot over to get useful information. Don't worry about the second case of the function in (III.2): Mathematica won't notice that the function is not defined if $x^2 + \sin(y)^2 = 0$.

(IV.3) Mathematica will (try to) solve systems of equations for you, using the `Solve` command. For example, to solve the system $x^2 + y^2 = 5$, $x^3 - y^3 = 7$, use:

```
Solve[{x^2 + y^2 == 5, x^3 - y^3 == 7}, {x, y}]
```

(Try it. The input format is a list of equations to solve, and then the list of variables to solve for. This can be very helpful for checking your work on Lagrange multiplier problems.)

(IV.4) The previous command gives two real solutions and a bunch of complex ones. If you want to get rid of the complex ones, you can tell Mathematica you're only interested in real solutions:

```
Solve[{x^2 + y^2 == 5, x^3 - y^3 == 7}, {x, y}, Reals]
```

(Try it.)

(IV.5) Mathematica will (try to) solve optimization problems for you, using Lagrange multipliers and other techniques. For example, to solve Problem 14.8.3 in the book, use

```
Minimize[{x^2+y^2, x*y==1},{x,y}]
Maximize[{x^2+y^2, x*y==1},{x,y}]
```

(Try it.)

(IV.6) Similarly, to solve Problem 14.8.21, use:

```
Maximize[{Exp[-x*y], x^2 + 4*y^2 <= 1}, {x, y}]
Minimize[{Exp[-x*y], x^2 + 4*y^2 <= 1}, {x, y}]
```

(Try it. The format is {function to minimize and list of constraints}, {list of variables to vary when maximizing}. You are encouraged to use Mathematica to check your work on the written and WebAssign problems.)

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