MATH W4051 PROBLEM SET 8
DUE OCTOBER 27, 2009.

INSTRUCTOR: ROBERT LIPSHITZ

(1) Munkres 51.1
(2) Munkres 51.2
(3) Munkres 51.3
(4) Let Homeo(X) denote the set of homeomorphisms $f\colon X \to X$.
   (a) Explain briefly why Homeo(X) is a group. Explain briefly how Homeo(X) acts on X. (If you've forgotten, look up what "acts on" means in a book on group theory.)
   (b) Show that for any nonempty spaces X and Y there is an injective group homomorphism Homeo(X) $\to$ Homeo($X \times Y$).
   (c) Show that Homeo(S^1) acts *transitively* on S^1. That is, show that for any points $x, y \in S^1$ there is an element $\phi \in$ Homeo(S^1) so that $\phi(x) = y$. (Hint: this should be easy.) Conclude that Homeo(S^1) is uncountable.
   (d) Let X be the union of the x- and y-axes in $\mathbb{R}^2$, with the subspace topology (so X is shaped like an X). Prove that Homeo(X) does *not* act transitively on X. (This takes a little insight.)
   (e) Find a space X with at least 3 points so that Homeo(X) is the trivial group.
   (f) Optional, challenge problem: can you find an infinite subspace X of $\mathbb{R}^n$ such that Homeo(X) is the trivial group?

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