The hypoelliptic Laplacian

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The hypoelliptic Laplacian ia a family of operators, indexed by $b \in \mathbf{R}^*_+$, acting on the total space of the tangent bundle of a Riemannian manifold, that interpolates between the ordinary Laplacian as $b \to 0$ and the generator of the geodesic flow as $b \to +\infty$. Up to lower order terms, the hypoelliptic Laplacian is a geometric version of a Fokker-Planck operator. It is not self-adjoint, it is not elliptic, it is hypoelliptic.

There is a probabilistic counterpart to the hypoelliptic Laplacian, which is a 1-parameter family of differential equations, known as geometric Langevin equations, that interpolates between Brownian motion and the geodesic flow.

In the case of locally symmetric spaces, the spectrum of the elliptic Laplacian remains rigidly embedded in the spectrum of its hypoelliptic deformation.

In this talk, I will describe the hypoelliptic Laplacian, and some of the applications obtained so far.