Columbia Mathematics Department Colloquium

$\mathrm{GL}_n(x)$ where x is an indeterminate ?

by

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Abstract:

Let $\mathrm{GL}_n(q)$ be the group of invertible $n \times n$ matrices with entries in the finite field with q elements. The order of $\mathrm{GL}_n(q)$ is the value at $x = q$ of the polynomial

$$x^{\binom{n}{2}} \prod_{i=1}^{i=n} (x^i - 1)\,.$$

We shall explain how not only orders of "natural subgroups", but also Sylow theorems, dimensions of irreducible (complex) representations, and even modular representation theory (representations in nonzero characteristic) of $\mathrm{GL}_n(q)$ may as well be described by polynomials evaluated at q. As if there were an object "$\mathrm{GL}_n(x)$" which would specialize to $\mathrm{GL}_n(q)$ for $x = q$.

Same phenomena occur for other finite groups of Lie type over finite fields (groups attached to Weyl groups of type B_n, D_n, $E_{m(m=6,7,8)}$, F_4, G_2).

It is then natural to try to construct similar polynomial data attached to other reflection groups, and even to groups generated by pseudo-reflections : this is the program named "Spetses". If time permits, we shall say a few words about it.

Wednesday, February 22nd, 5:00 - 6:00 p.m.

Mathematics 520

Tea will be served at 4:30 p.m.