ABSTRACT

Contributions to the Theory of Optimal Stopping

for One–Dimensional Diffusions

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We give a new characterization of excessive functions with respect to arbitrary one–dimensional regular diffusion processes, using the notion of concavity. We show that excessive functions are essentially concave functions, in some generalized sense, and vice–versa.

This, in turn, allows us to characterize the value function of the optimal stopping problem, for the same class of processes, as "the smallest nonnegative concave majorant of the reward function". In this sense, we generalize results of Dynkin–Yushkevich [2] for the standard Brownian motion. Moreover, we show that there is essentially one class of optimal stopping problems, namely, the class of *undiscounted* optimal stopping problems for the standard Brownian motion. Hence, optimal stopping problems for arbitrary diffusion processes are not inherently more difficult than those for Brownian motion.

The concavity of the value functions also allows us to draw sharper conclusions about their smoothness, thanks to the nice properties of concave functions. We can therefore offer a new perspective and new facts about the smooth–fit principle and the method of variational inequalities in the context of optimal stopping.

The results are illustrated in detail on a number of non–trivial, concrete optimal stopping problems, both old and new.