GROUPS AND REPRESENTATIONS II: SYLLABUS

- Survey and History of Representation theory; Generalities about Quantization and Representation Theory

- Lie Algebras and Their Relation to Lie Groups

    - Exponential mapping and its Jacobian
    - Adjoint and co-adjoint representation
    - Baker-Campbell-Hausdorff formula

- Representations of Compact Lie Groups: Cartan-Weyl Highest Weight Theory

    - Review of Peter-Weyl theorem
    - Maximal Tori: Existence, Uniqueness up to Conjugation, every element is in a maximal torus. Examples for classical groups
    - Topology and geometry of G/T
    - Weyl group, action on maximal torus and on its Lie algebra
    - Roots: positive roots, dominant alcove
    - Dynkin diagrams and classification of root systems.
    - SU(2) representations and their applications in Physics
    - Weight spaces, dominant weights
    - Highest weight theorem
    - SU(3) representations
    - Weyl integral formula, character formula and dimension formula

- Homogeneous Vector Bundles

    - Induced representations
    - Frobenius Reciprocity
    - Spherical Harmonics

- Representations of Compact Lie Groups: Borel-Weil Theorem and Geometric Theory

    - Review of Peter-Weyl Theorem
    - Borel-Weil theorem
    - Examples, representations on homogeneous polynomials
    - Borel and parabolic subgroups, flag manifolds

- The Borel-Weil-Bott theorem

- Lie algebra cohomology, cohomology of Lie groups
- Borel-Weil-Bott theorem
- Weyl character formula and Euler characteristics

- Hamiltonian mechanics, symplectic geometry, geometric quantization and the orbit method

- Projective representations

- The Spinor Representation

  - Spin(2n) as a double cover of SO(2n)
  - Projective representations
  - The Clifford Algebra, Canonical Anticommutation Relations

- The Metaplectic Representation

  - The Heisenberg algebra and group, Canonical Commutation Relations
  - Stone-von Neumann Theorem.
  - The Metaplectic double cover of Sp(2n) and the Metaplectic Representation
  - Theta functions

- Correspondence between representations of GL(n) and S_n

- Kac Moody algebras, the Virasoro algebra and their highest weight representations(?)