Groups and Representations II: Problem Set 3
Due Monday, March 31

Problem 1:

Using the explicit representation of $SU(2)$ on homogeneous polynomials given in class, compute the action of the elements of a basis of the Lie algebra on these representations.

Problem 2:

Explicitly write out the Weyl integral and character formulas for $G = SU(3)$. For the case of the defining representation of $SU(3)$ on $\mathbf{C}^3$, check the Weyl character formula by comparing its result to the direct computation of the trace.

Problem 3:

Characterize the irreducible representations of $SU(n)$ in terms of $n-1$ non-negative integers, the coefficients of the highest weight, expressed in terms of the basis of fundamental weights.
Use the Weyl dimension formula to give a dimension formula for these representations. In particular, for $SU(3)$, what are the dimensions and highest weights of all irreducible representations of dimension less than 16?

Problem 4: Complete the proof of Frobenius reciprocity outlined in the notes.

Problem 5: Let $G = SU(4)$. Explicitly identify each of the distinct parabolic subroups $P \subset G_{\mathbf{C}}$ and how it is related to nodes of a Dynkin diagram. For each such P, give a geometrical definition of the space $G_{\mathbf{C}}/P$ and find the group H such that $G_{\mathbf{C}}/P = G/H$.

Problem 6: For $G = SU(3)$, explicitly define an infinite sequence of irreducible representations on a space of homogeneous polynomials. Compute the highest weights and dimensions of these representations.