Modern Geometry, Fall 2017: Problem Set 6

Due Thursday, October 19

Problem 1: Show that every symplectic manifold is orientable. Hint: consider the top exterior power of the symplectic form.

Problem 2:

Show that the Stokes and divergence theorems of vector analysis in $\mathbf{R}^3$ are special cases of the general Stokes theorem using differential forms discussed in class.

Problem 3: Show that $SO(3)$ is diffeomorphic to the real projective space $\mathbf{R}P^3$.

Problem 4: Show that $GL(n, \mathbf{R})^+$ (the orientation-preserving component of $GL(n, \mathbf{R})$) is diffeomorphic to $SO(n) \times \mathbf{R}^{\frac{n(n+1)}{2}}$. Hint: use the Gram-Schmidt algorithm to get the relation between an arbitrary basis of $\mathbf{R}^n$ and an orthonormal basis.