Modern Geometry, Fall 2017: Problem Set 3
Due Thursday, September 28

Problem 1: If α is a smooth tensor field in $\Gamma(T^{k,l}M)$ on a smooth manifold M, and x^i are $\tilde{x}^i$ are two sets of coordinates corresponding to two different coordinate charts on some open set $U \subset M$, find the formula relating the coefficient functions for α with respect to the two coordinate bases.

Problem 2: Prove that the tensor product defined in class on multilinear functions is associative.

Problem 3: Prove that a covariant k-tensor α is antisymmetric iff $Alt(\alpha) = \alpha$

Problem 4: Given a smooth map $F : M \to N$ between smooth manifolds M and N, show that the pull-back map $F^* : \Omega^*(N) \to \Omega^*(M)$ satisfies
$F^*(\alpha \wedge \beta) = F^*\alpha \wedge F^*\beta$ for $\alpha, \beta \in \Omega^*(N)$