Modern Geometry II: Problem Set 4

Due Monday, May 2

Problem 1: Consider the complex manifold $M = \mathbf{C}P^n$, and the map

$$\pi : \mathbf{C}^{n+1} - 0 \to \mathbf{C}P^n$$

given by taking the complex line through the origin and a point in $\mathbf{C}^{n+1} - 0$. Use the function

$$\ln(\sum_{i=1}^{n+1} |z_i|^2)$$

on $\mathbf{C}^{n+1} - 0$ as a Kähler potential and calculate:
a) The Kähler form on $\mathbf{C}P^n$.
b) The associated Kähler metric on $\mathbf{C}P^n$. This is known as the Fubini-Study metric.

Problem 2: a) Show that $Cliff(3) = \mathbf{H} \oplus \mathbf{H}$.

b) Show that Spin(3)=SU(2).

Problem 3: In an orthnormal frame $\{e_1, e_2, \cdots, e_n\}$ the Dirac operator on $\Gamma(S)$ is

$$D = \sum_{i=1}^{n} e_i \cdot \nabla_{e_i}$$

where the first e_i acts by Clifford multiplication. Show that this formula for the Dirac operator doesn't depend on the choice of orthonormal frame.