Groups and Representations II: Problem Set 3

Due Monday, Monday March 19

Problem 1: Hall, Problem 5 of Chapter 7, page 240

Problem 2: Derive the Kostant multiplicity formula and work out an example for SU(3). More explicitly, do Sepanski Exercise 7.21, page 175.

Problem 3: For $G = SU(3)$, explicitly define an infinite sequence of irreducible representations spaces of homogeneous polynomials. Use the Borel-Weil theory to describe these geometrically and relate them to their highest weights. Use the Weyl dimension formula to compute the dimensions of these representations.

Problem 4: Do

Hall, Problem 6 of Chapter 7, page 240.
OR
Sepanski, Exercise 7.34, page 185.

Problem 5: Show that the real Clifford algebra for $\mathbf{R}^3$ is isomorphic to the sum of two copies of the quaternion algebra, i.e.

$$C(3) = \mathbf{H} \oplus \mathbf{H}$$