Problem 1:
Show that a group $G$ (for which the convolution product is defined) is commutative iff the convolution product is commutative.

Problem 2:
For a representation $V$ of $G = SU(2)$, show that the multiplicity of the $n$'th irreducible representation in $V$ is given by the coefficient of $z^{n+1}$ in

$$(z - z^{-1})\chi_V(z)$$

Problem 3:
Using characters, find the decomposition into irreducibles of the tensor product $V_n \otimes V_m$ of irreducible representations of $SU(2)$.

Problem 4:
Let $G = SU(n)$, and $T$ the subgroup of diagonal elements. Consider the map

$$\pi: G/T \times T \to G$$

given by $\pi(gT, t) = gtg^{-1}$ What is the Weyl group $W$ in this case? Show that this map is a $|W|$-fold covering away from a locus in $G$ of dimension less than $\dim G$. What is this locus of points in $G$ where $\pi$ is not a $|W|$-fold covering?

Problem 5:
For $G = SO(2n)$, identify a maximal torus and the positive roots. Give an explicit version of the Weyl integral formula in this case as an integral over this maximal torus.

Problem 6: For $G = SO(3)$, identify a maximal torus $T$, the space $G/T$, and the Weyl group $W$. Give an explicit construction of the irreducible representations of $G$, compute their characters, and use the Weyl integration formula to show that they are orthonormal.