Introduction to Quantum Mechanics, Fall 2012: Syllabus

- Introduction
  - Basic axioms of quantum mechanics
  - Unitary group representations
- The group $U(1)$ and charge
  - Some representation theory: Schur's lemma, representations of commutative groups
  - The group $U(1)$ and its representations
  - Charge and quantum mechanics
- Two-state quantum systems: the spin-1/2 particle/the qubit
  - Pauli matrices and observables of two-state systems
  - The groups $U(2)$, $SU(2)$ and their Lie algebras
  - The spin 1/2 particle in a magnetic field
- Review of linear algebra
  - Hermitian vector spaces, dual vector spaces
  - Linear operators, change of basis
  - Orthogonal and unitary groups
  - Eigenvalues, eigenvectors
- Lie algebras and Lie algebra representations
- Rotations and spin groups in three and four dimensions
- The spin representation, the spin 1/2 particle in a magnetic field
- Classification and construction of representations of $SO(3)$ and $SU(2)$, angular momentum operators
- Tensor products of representations and entanglement, addition of angular momentum
- The group $\mathbf{R}$ and time translations
  - The Hamiltonian, Schrödinger equation
  - Basics of Fourier analysis
  - Energy eigenvectors and eigenstates

- The group $\mathbf{R}^3$ and space-translations. The momentum operator
- The Heisenberg and symplectic groups
  - Phase space, Hamiltonian mechanics, Poisson brackets
  - The Heisenberg commutation relations
  - Generalities about quantization
  - The Stone-von Neumann theorem
  - The Groenwald-van Hove theorem
- The harmonic oscillator
- Particle in a potential
  - Central potentials
  - The isotropic harmonic oscillator in $\mathbf{R}^3$
  - The hydrogen atom, $SO(4)$ symmetry and group theoretical methods for solution
- Continuing on to a second semester, topics may include: Path integrals, fermionic variables, special relativity, gauge symmetry and particles in an electromagnetic field, the Dirac equation, quantization of fields, quantization of the electromagnetic field, decoherence and measurement theory, quantum information theory.