Lie Groups and Representations: Syllabus

Fall Semester

- Survey and history of Lie groups and representation theory; generalities about quantization and representation theory
- Lie groups and Lie algebras: some examples of closed linear groups
- Review of some differential geometry
- Lie algebras and their classification
- Finite dimensional representations of $sl(2, \mathbb{C})$
- Complex semisimple Lie algebras
  - Cartan subalgebras
  - Roots and root systems
  - Weyl groups
  - Classification
- The universal enveloping algebra, Poincare-Birkhoff-Witt theorem
- Finite-dimensional representations, highest-weight theory
  - Highest-weight theorem
  - Verma modules
  - Harish-Chandra homomorphism, the infinitesimal character
  - Weyl character formula, dimension and multiplicity formulae

Spring Semester

- Generalities about representation theory of finite groups
- Review of Fourier analysis, representations of abelian groups
- Representations of compact Lie groups
  - Peter-Weyl theorem
  - Maximal tori
  - Weyl integral formula, character formula and dimension formula
  - Topology and geometry of $G/T$
  - Borel and Parabolic Subgroups, flag manifolds
  - Induced representations and Frobenius reciprocity
– Borel-Weil theorem
– Examples, representations on homogeneous polynomials
– Applications of SU(2) and SU(3) representations in physics

• Hamiltonian mechanics, symplectic geometry, geometric quantization and the orbit method

• The Spinor Representation
  – Spin(2n) as a double cover of SO(2n)
  – The Clifford Algebra, Canonical Anticommutation Relations

• The Metaplectic Representation
  – The Heisenberg algebra and group, Canonical Commutation Relations
  – Stone-von Neumann Theorem.
  – The Metaplectic double cover of Sp(2n) and the Metaplectic Representation
  – Theta functions

• Correspondence between representations of GL(n) and S_n

• Structure theory of semisimple groups
  – Iwasawa decomposition
  – Real forms

• Representations of a non-compact semi-simple group: SL(2, \mathbb{R})

• Modular forms, Hecke algebras and (very optimistically) an extremely basic introduction to the Langlands program

• Other possible topics
  – Lie algebra cohomology and the Borel-Weil-Bott theorem
  – Kac Moody algebras, the Virasoro algebra and their highest weight representations