Lie Groups and Representations: Syllabus

Fall Semester

- Survey and history of Lie groups and representation theory; generalities about quantization and representation theory
- Lie groups and Lie algebras: some examples of closed linear groups
- Review of some differential geometry
- Lie algebras and their classification
- Finite dimensional representations of $\mathfrak{s}l(2, \mathbf{C})$
- Complex semisimple Lie algebras
  - Cartan subalgebras
  - Roots and root systems
  - Weyl groups
  - Classification
- The universal enveloping algebra, Poincare-Birkhoff-Witt theorem
- Finite-dimensional representations, highest-weight theory
  - Highest-weight theorem
  - Verma modules
  - Harish-Chandra homomorphism, the infinitesimal character
  - Weyl character formula, dimension and multiplicity formulae

Spring Semester

- Generalities about representation theory of finite groups
- Review of Fourier analysis, representations of abelian groups
- Representations of compact Lie groups
  - Peter-Weyl theorem
  - Maximal tori
  - Weyl integral formula, character formula and dimension formula
  - Topology and geometry of G/T
  - Borel and Parabolic Subgroups, flag manifolds
  - Induced representations and Frobenius reciprocity

  - Borel-Weil theorem
  - Examples, representations on homogeneous polynomials
  - Applications of SU(2) and SU(3) representations in physics

- Hamiltonian mechanics, symplectic geometry, geometric quantization and the orbit method

- The Spinor Representation
  - Spin(2n) as a double cover of SO(2n)
  - The Clifford Algebra, Canonical Anticommutation Relations

- The Metaplectic Representation
  - The Heisenberg algebra and group, Canonical Commutation Relations
  - Stone-von Neumann Theorem.
  - The Metaplectic double cover of Sp(2n) and the Metaplectic Representation
  - Theta functions

- Correspondence between representations of GL(n) and S_n

- Structure theory of semisimple groups
  - Iwasawa decomposition
  - Real forms

- Representations of a non-compact semi-simple group: $SL(2, \mathbf{R})$

- Modular forms, Hecke algebras and (very optimistically) an extremely basic introduction to the Langlands program

- Other possible topics
  - Lie algebra cohomology and the Borel-Weil-Bott theorem
  - Kac Moody algebras, the Virasoro algebra and their highest weight representations