GROUPS AND REPRESENTATIONS I: PROBLEM SET 4
Due Monday, November 12

Problem 1: Prove that, for each root α, the root-space L_α has dimension one.

Problem 2: For L the Lie algebra of $GL(n, \mathbf{C})$, show that the Killing form is given by
$$B(X_1, X_2) = c_1 Tr(X_1 X_2) - c_2 Tr(X_1) Tr(X_2)$$
for some constants c_1, c_2, and find the values of these constants.

Problem : If one has a choice of a basis Π of positive roots and $\alpha \in \Pi$, show that each reflection s_α permutes the set of positive roots other than α, and that the entire Weyl group W is generated by these s_α.

Problem 4: Knapp Problem II.10

Problem 5: Knapp Problem II.15

Problem 6: Knapp Problem II.17 (Given that one can reconstruct a Lie group from a root system, this problem shows how to find the "dual group" to a given Lie group).